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DETECTION OF TRANSIENTS IN NUCLEAR SURVEILLANCE-COUNTING CHANNELS

by

K. G. Porges

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K. G. Porges

Reactor Physics Division

November 1968

"System analysis is when you ought to be maximizing your decision reliability by computerizing all contingencies, but the breadth of problems and lack of data mean you have to make up all the figures out of your head."

Frederick Crews, "The Patch Commission"

TABLE OF CONTENTS

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF FIGURES

LIST OF TABLES

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ABSTRACT

In certain nuclear-instrumentation problems, one must detect short excursions in a normally quasi-constant input count rate. This report reviews a number of such "transient detection" systems and describes in some detail one particular application, fuel-failure monitoring in connection with high-power density reactor cores. The instrument generally incorporates a "filter" which must be arranged to optimize the display of the transient, or (where the channel output is used to actuate an alarm) to result in a specified detection efficiency and acceptable false-alarm frequency, in conjunction with an acceptable announcement delay.

This report discusses the design of such a filter and gives formulas for estimating the false-alarm frequency at a specified detection efficiency. An elementary formula turns out to be of dubious reliability when the normal background rate is low enough to yield only very few pulses within a time-span defined by the "memory" of the filter. A more precise formula is derived for such cases.

Where the false-alarm frequency cannot be reliably reduced to an acceptable value and several independent detector channels are available, certain logic networks can yield improvements. A number of such logic interfaces are discussed and appropriate false-alarm frequencies derived.

In connection with the optimum filtering problem, the beneficial effect of pulse-shape symmetrization is considered in detail. A simple digital channel is described which should provide improved performance parameters with considerably better long-term reliability than conventional analog equipment.

I. INTRODUCTION

In general terms, the detection of transients in a surveillance or search system implies timely warning that a normally rare event of a particularly interesting or possibly dangerous nature has occurred. Examples of this type of situation abound in many fields of science and technology, some of which are not readily discussed in the open literature. The subject of this report may thus have been considered in many different contexts in specialized or classified reports. More general treatments have chiefly emphasized radar applications.[1] The statistics of count channels, on the other hand, are dealt with in a thorough but limited vein in several works on stochastic processes,[2-5] as well as more summarily in books and articles on nuclear instrumentation, where the operation of count-rate meters is usually discussed without consideration of response to short transients. Somewhat more specific treatments of the false-alarm problem in a nuclear-surveillance channel have appeared,[6] based on the well-known treatment of the zero-crossing rate of random noise by Rice.[7]

A. General Description of Event-rate Transient Monitor

In a typical transient-detection system of the type considered in this report, pulses of short duration are obtained from one or more nuclear-radiation detectors which scan a source of such radiation continuously. Under normal conditions, the source is assumed to vary relatively slowly in time or remain constant but to increase strongly for a relatively short time when the event to be detected occurs. The channel count rate thus consists of a certain quasi-constant background with a superposed transient signal. Other extraneous background due to unwanted detector pulses or electronic noise can usually be removed by pulse-height discrimination.*

The pulses delivered by the pulse-height discriminator, still short but of standard height and duration, are then fed to a processor, which converts this digital input into an analog output. In the absence of a transient, the output fluctuates in a way determined by the response of the processor to the Poisson fluctuations of the input count rate. A transient (which may have a certain characteristic structure) is correspondingly distorted by the processor.

In almost any of the applications of transient monitoring mentioned in Section II (to which, no doubt, many other examples could be added), the expected transients may be much more intense than the background.** The reliability of detection may then be readily made so high that any discussion of filter response or statistics of detection would be entirely superfluous. For other situations, however, transients, somewhat less intense in comparison with background, may occur with different shapes, from which something can be learned regarding the source. It is then evidently profitable to investigate filter response with a view of emphasizing shape differences

*Where there is no extraneous background problem, the nuclear-radiation detector may be arranged to deliver a current directly into a wide-passband amplifier, which thus becomes the signal processor. This arrangement exhibits essentially the same statistical behavior as the pulse system described here.

**As a typical example, the background may be due to "dark current" in a photomultiplier, while the transients are short but relatively intense light flashes, as the photomultiplier scans a perforated mask over a light source.

while minimizing random background fluctuations, so that the recorded output may more readily reveal the "signature" of certain events. In many transient-detection applications, the attention of operating personnel is usually engaged by other instruments. This situation calls for an alarm (of some sort), which goes into action whenever the analog output exceeds a certain set level. Similarly, in an area scan (as in the example given earlier) the transients essentially actuate an alarm, which registers the coordinates through some mechanism coupled to the drive.

B. Performance Parameters of Alarm System

The use of an alarm discriminator makes the performance of the system amenable to quantitative evaluation in terms of statistical or information theory. The detection probability, for example, can be calculated from the set level and filter response, to the extent that the intensity and duration of the transient can be reliably estimated. Inevitably, the level will be occasionally exceeded by a large background fluctuation; the corresponding false-alarm frequency can also be calculated. (This does not include such other sources of noise as electrical pickup, intermittent breakdown, and other malfunctions, which may often be a major source of difficulty, but cannot be considered as inevitable.) A third performance index is the speed of detection or corresponding announcement delay between event and alarm. This parameter, which depends on the alarm level, transient intensity, and filter response, is important in high-speed area scanning, where it can introduce a speed-dependent slewing of the coordinate. It is even more obviously important in safety-type monitoring, where an event that gives rise to a transient could be connected with the onset of serious trouble.

These three performance parameters (detection probability, false-alarm frequency, and announcement delay) are evidently interrelated and together depend on the three input variables: mean background rate, transient intensity, and transient duration, as well as on the instrument parameters: filter frequency response and alarm level. The performance parameters are moreover limited to a certain range through practical considerations; this simplifies somewhat the task undertaken in this report of finding approximate equations which describe the performance of transient-detection systems. Regarding the detection probability, for example, one may safely exclude efficiencies below, say, 90%. On the other hand, detection probabilities over 99% are demanded only for some special missions and can often be achieved only by allowing a rather high false-alarm frequency. The latter parameter is somewhat dependent on the possible deployment of several transient-detection systems (of which one, used for timely warning, may have a rapid response, while others, through partial integration, may have an inherently lower false-alarm frequency and at the same time slower response). Aside from such considerations, the general rule is that the product of false-alarm frequency and mean operating time of the system must be much less than the number of transients (for area scanning) or somewhat less than unity (for safety monitoring). For scanning systems, the announcement delay must be somewhat smaller than the duration of the transient. For safety monitoring, this delay can be defined only with some knowledge of the mechanisms through which dire consequences may develop from the cause of the transient. Since such knowledge is often not available, the detection delay may generally be set at a time not exceeding other delays in the overall system (e.g., mean transport times from core to detector in reactor safety installations).

In this report, performance parameters will be assumed to be confined within these practical limits. To avoid undue complexity, we use the simplest transient model, an abrupt increase in count rate of uniform intensity, ceasing abruptly after a time T. In the same vein, only the simplest, readily implemented filters are treated explicitly. (The discussion includes, however, a treatment of logic processing of several channels.) With these rudimentary model assumptions, straightforward relations between the performance parameters are obtained which one expects to be valid in the limit of very large values of the statistical parameter nT_b (n = normal background rate, T_b = representative filter time constant). Since, furthermore, certain criteria for transient detection are optimized when $T_b \approx T$ = transient duration, the above-mentioned limiting condition amounts to the stipulation that N, the number of background counts during the transient, be large. This condition may be met in certain favorable cases of transient detection, but can only be approximately satisfied in some of the more important applications. For these cases,* there is, strictly speaking, no reliable treatment available, and the relations become approximations that suggest, rather than prescribe, suitable choices of filter constants and alarm level.

In the organization of a theoretical work aiming at specific practical applications, one has the choice of (1) presenting all equations and then discussing their application and validity in various practical situations, or (2) inserting practical digressions while developing the mathematical framework. The latter approach is followed here, mainly to clarify the difficulties encountered in applying the equations just where they would be particularly helpful. The practical example of transient detection most frequently referred to, which provided the original motivation for this work, is a particular type of reactor fuel failure monitor, the FERDL (Fuel Element Rupture Detection Loop) of EBR-II, described in some detail in Refs. 8 and 9. Features of this system having a specific bearing on transient detection are discussed in Section II.

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*The background rate may be low precisely because strenuous efforts have been expended to make it so. The fact that this also renders the application of statistical theory questionable should not be interpreted as a denigration of such efforts; rather, it stresses the desirability of increasing the overall sensitivity, which may increase the background rate, but will increase the signal count even more.

II. SPECIFIC TRANSIENT-DETECTION SYSTEMS

The following brief survey of equipment systems that can be classified as transient detectors is limited in principle to instrumentation based on particle or photon counters. Thus, the input is a series of counts as described in Section I. Within that broad category, one may then further distinguish between area surveys and stationary transient detectors.

A. Area-survey Systems

Systems belonging to the area-survey group have the task of locating stationary "sources" distributed over a certain area or perhaps volume. To that end, a detector, equipped with a collimator or mask (and possibly accompanied by a primary source that stimulates emission), is made to move in a pattern that covers the area. At the same time, the coordinates of the mask are kept track of; coordinate registration is triggered by the "alarm" device coupled to the transient detector. The relative intensities and spatial width of individual transients may also be required, whereas the detailed shape of transients usually conveys no useful information. Although an exhaustive listing of examples of this kind of system would be beyond the scope of this report, some typical applications come readily to mind. First, there are surveying systems physically covering the earth's surface or subsurface, such as mineral surveys, oil-well logging, mine detection, or other military applications. Second, there are laboratory systems in which a small object or photograph is scanned; this includes in vivo scanning, track-plate scanning, cloud or bubble-chamber photograph scanning, and similar tasks. Finally, there may be applications in astronomy and allied fields. All these systems have a common characteristic: Direct relations exist between signal duration and sweep speed on the one hand, and resolution and sweep speed on the other.

In favorable situations, the contrast between presence and absence of the "source" may be so strong at the highest practical sweep speed that there is no need to appeal to statistical theory. In still other practical situations, only qualitative information may be sought. Hence, a rather primitive statistical theory would be sufficient; areas of special interest may be then covered with repeated sweeps until a clear picture emerges. However, it is also conceivable that the area to be covered is large, while very good resolution is required and the sweep speed can be readily increased. In that case, the detection probability and false-alarm frequency for a certain sweep speed may well be of more than academic interest.

B. Transient Detection for Warning or Safety

Turning now to the second group of transient detectors, we find stationary systems largely employed for protection; transients thus imply danger and call for some sort of remedial action. The information sought often includes the "signature" or detailed shape of the transient, which is--or at least is hoped to be--a rare event. In contrast to the systems considered above, there is no control over transient duration, nor can one return for a second look. On the other hand, there may be more scope for elaborate and heavy instrumentation than for a vehicular system; for example, detectors can be readily multiplied up to a fairly generous limit. As a single but important practical example of such a system, we shall consider detection of fuel-cladding failure in nuclear power plants.

In the first group of transient detectors, the required sweep speed played a decisive role in determining the relative significance of statistical concepts for a given application. For fuel-failure detection, a similar role is played by the announcement delay allowed by the presumed consequences of a fuel failure. Thus, certain types of reactors tend to develop small leaks in the cladding, which do not immediately endanger the plant, merely resulting in the contamination of the coolant by fission products. One may then use detection systems that accumulate fission products from the coolant in some way, thus increasing sensitivity of detection by sacrificing speed. Such detectors are not actually transient detectors and can be designed and operated without applying statistical theory. In contrast, other types of reactors do not exclude the possibility of more violent cladding failures, which could conceivably result in a rapid proliferation of damage. Such plants evidently call for transient detection.

Specification of the maximum allowable announcement delay, as with many other considerations connected with safety, is largely a matter of policy. The formulation of this policy must consider (among other factors) the likelihood of certain kinds of cladding failure for a given type of fuel and the further likelihood of serious damage resulting from such failures. For the more advanced reactors, such estimates can be supported only by rather limited experience and test results; however, in general the speed of failure detection desirable for a given reactor design is directly correlated with the power density. To argue this point, we suppose that a "catastrophic" failure of one fuel pin is conceivable which blocks adjacent coolant flow channels with debris. At a power density of the order of 1 kW/cm^3, typical of fast, sodium-cooled breeders, such a failure would entail a predictable, rapid, local temperature rise, resulting in a highly probable rapid proliferation of damage. In contrast, a similar incident in a water-cooled thermal-spectrum reactor plant, operating at much lower power density, would have a much smaller likelihood of seriously damaging adjacent elements.

Effective fuel-failure monitoring for fast breeders (such as EBR-II) must thus provide for fast warning in at least some of the equipment channels. In addition, other channels, which integrate the signal and thus can be inherently more sensitive, are useful for diagnostic purposes. With several channels, based on different principles, it should be possible to draw inferences regarding a failure that does not emit debris from the relative amount of short- and long-lived fission products, the relative amount of fission products in the coolant and in the gas blanket, etc. The specialization of this report in transient detection is not intended to de-emphasize the importance of other, integrating monitor channels. However, such channels do not provide the speed of detection that could, in principle, forestall serious damage to the core in the event (however unlikely) of a debris-emitting cladding rupture.

C. The FERD System

To describe such a transient monitor in some detail, we consider the FERD system, installed at the EBR-II plant, as an example. The driver fuel of this fast breeder consists of uranium alloy pins bonded to stainless jackets by an annulus of sodium; the cladding further contains a certain amount of argon. The bonding sodium is normally enriched with short-lived fission products, inter alia delayed-neutron precursors; longer-lived, highly volatile fission products tend to collect in the internal gas cover. Assuming that a serious cladding burst occurs, one can predict that the bonding sodium will be rapidly expelled and mixed with coolant sodium; in addition, debris of the fuel and cladding (or droplets of molten fuel) may be rapidly injected into the coolant. This contamination is carried through the upper plenum and through the heat exchanger, where the flow is sampled. The degree of mixing and the sampling efficiency are not known and may be suspected to vary considerably with the location of the burst can in the core. The high flow speed of this plant, however, generally prevents widespread dispersion of contamination, which should therefore enter the sampling loop as a spike or "slug" of highly contaminated coolant, followed by a "tail" of less contamination due to direct emission of fission products from bare fuel remaining in the core. (The "tail effect" depends on the degree of channel blockage.) The monitoring loop pumps a fraction of the coolant stream past a bank of neutron detectors, then back into the main coolant tank. A slug of delayed-neutron precursors resulting from a burst as described above can be estimated to pass the detector bank in 0.2-0.8 sec. Once returned to the tank, the activity is dispersed in a very large volume of coolant and cannot return to the loop until the short-lived delayed-neutron emitters have decayed through many half-lives. Volatile fission products, however, gradually pass into the gas blanket, where the more slowly decaying species among them can be detected by means of an integrating monitor of high sensitivity and reliability.

The strength of the transient signal can be reasonably inferred from calculations of equilibrium enrichment of bonding sodium at full reactor power, supposing further that one-third to two-thirds of the bonding sodium is rapidly expelled through the pressure of the internal gas blanket of the fuel pin, and assuming representative sampling.[10] The detection efficiency is known from a calibrated neutron test source; the overall detection efficiency is also available from a series of tests in which a bare piece of fuel of known surface was inserted in the core.[11]

Except perhaps for the uncertain effectiveness of the sampling snout, the EBR-II FERD system may thus be regarded as a fairly straightforward example of a transient monitor whose input parameters can be inferred reliably enough to provide a practical example for the theoretical relations between design and performance parameters discussed in this report.

These matters are now taken up in detail, beginning in Section III with the conversion of the digital input pulse train into an analog signal.

The above description of the FERD system does not include the means provided for display of transients. For the sake of completeness, these are briefly described in Appendix E.

III. THEORETICAL RESPONSE OF THE PROCESSOR TO MODEL TRANSIENTS

The train of digital pulses delivered by the nuclear-detector channel at the front end of any of the transient detectors described in Section II is converted into an analog signal through a "processor" consisting of several filter circuits, of which the count-rate meter forms the first. The object of filtering is to deliver a clearly readable trace, in which (1) true transients are readily distinguished from statistical excursions of the background and (2) any features characteristic of a particular type of transient are clearly portrayed.

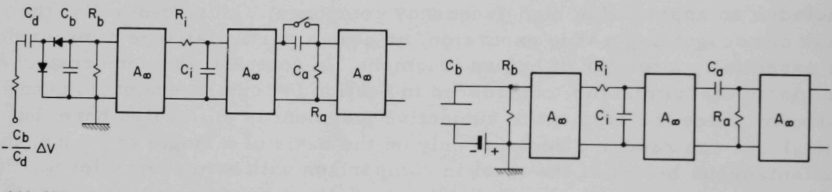
As regards the first-named requirement, this can be described quantitatively through the signal-to-noise (S/N) ratio, exactly as in pulse amplifiers, in which connection the optimum filtering problem has been widely discussed. The second criterion is less amenable to quantitative treatment, in view of the difficulty of specifying the characteristic shape or "signature" of the transient. Strong integration, that is, removal of high-frequency components, will make it difficult to recognize a signature that includes an appreciable high-frequency component. More generally, the task of recognizing a true excursion, or even a particular type of excursion, is essentially a matter of human judgment. In contrast, the performance of an alarm discriminator, considered in Section IV, can be evaluated quantitatively, since the element of subjective judgment is eliminated here; but the alarm can render a decision only on the basis of a single criterion, the instantaneous height of the level in comparison with a reference level. For a transient detector whose verdict is crucially important and which operates under difficult conditions (such as, perhaps, a fuel-failure monitor), it may therefore be expedient to provide an alarm only for alerting personnel and thereafter relying for a final decision on interpretation of the trace.

The response of the processor may be fully described in the time domain through its response function, $H(T,t)$, to a transient of duration T, consisting of a train of individual pulses of comparatively short duration, τ. The response, $F(t)$, to any single pulse is a special case of $H(T,t)$; alternatively, $H(T,t)$ can be considered as derived from $F(t)$ through convolution, as discussed, for example, in Ref. 1. The performance of the processor may be equally well specified in the frequency domain, which is a more natural mode of description for pulse amplifiers subject to different types of noise characterized by different frequency spectra. For the present case, the time domain appears more appropriate and will be adhered to in the discussion here. As regards the filter circuits, an increasing number of fairly complex circuits, including active and passive types, delay-line differentiators, etc., is now coming into use in connection with pulse amplification. For the time regime appropriate for transient detection, however, most of these filters are impractical or difficult to construct. The discussion below is thus limited to the most elementary R-C networks.

A. Count-rate Meter Circuit

A brief discussion of the operation of a count-rate meter for purposes of transient detection is warranted; although the basic facts concerning this type circuit are well known, they are usually considered only in connection with slowly changing count rates.

Many versions of the basic Count-Rate Meter (CRM) circuit, shown schematically in Fig. 1a, have been developed for a wide variety of nuclear applications. Differing considerably in circuit detail, as well as in linearity, stability, and available options, most such instruments behave as the elementary circuit shown. In this circuit, standard pulses of very short length τ (typically 1 μsec) charge up a "dipper" capacitor, C_d, with a standard charge, q, which is transferred by means of a diode pump circuit to the "bucket" capacitor, C_b, whence it slowly bleeds away through resistor R_b. The situation closely resembles the input of a voltage-sensitive pulse amplifier coupled to an ionization chamber or junction detector in which short current pulses of roughly standard height are generated, say by alpha particles or fission fragments. This circuit is shown in Fig. 1b.



113-656

a. Basic CRM Circuit

b. Pulse-amplifier Circuit

Fig. 1. Circuit Arrangements for Transients and for Pulse Processing. A_∞ = ideal amplifiers (∞ passband) with arbitrary gain. The CRM is equipped with a diode pump at the input; the amplifier is fed by a detector, such as an ion chamber, delivering short current pulses of roughly constant height. Circuits, hence analysis or response, are similar, but parameters are different. The final clipper is used in the CRM circuit only for transient monitoring; the final amplifier feeds an alarm discriminator.

The output signal, F(t), of the CRM in response to a single input pulse rises almost linearly during τ and subsequently decays exponentially. That is,

$$F(t) = (\Delta V/x)\left(1 - e^{-t/T_b}\right) \approx \Delta Vt/\tau; \qquad 0 \le t \le \tau, \tag{1a}$$

$$= (\Delta V/x)(e^x - 1)\, e^{-t/T_b}, \qquad \tau \le t; \tag{1b}$$

where

$$\Delta V = q/C_b, \tag{2}$$

$$T_b = R_b C_b, \tag{3}$$

and

$$x = \tau/T_b. \tag{4}$$

The step height, ΔV, is adjustable in most CRM's through a number of choices of $C_b = q/\Delta V$; the response, V, to a steady input rate n is adjustable through choice of $R_b = V/nq$. Figure 2 shows F(t) and the input pulse on two different time scales.

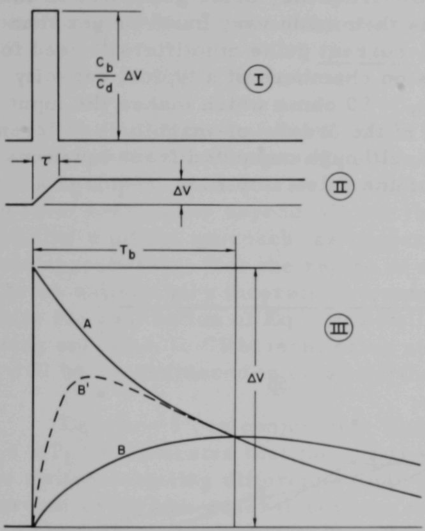


Fig. 2

Single-pulse Response, F(t), of Circuit Shown in Fig. 1a. The input is shown in I with the diode bridge disconnected; the pulse appearing at the first stage is shown in II. The same picture is shown in III, on a shortened time scale and increased voltage scale, Curve A; Curves B and B' are single-pulse responses at the entrance of the second stage (i.e., after passing filter R_iC_i). Curve B' represents a short time constant $T_i << T_b$; Curve B is drawn for $T_i = T_b$.

113-650

Suppose now that a sudden transient in the count rate steps up the rate of delivery at C_b for a time T of about 0.1-1 sec, and then ceases equally abruptly, so that a mean number of pulses M, in addition to the background, nT, are delivered during T. The circuit response to such a transient is then given by

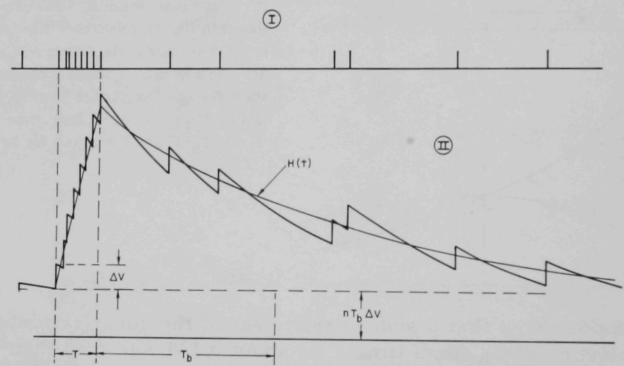
$$H(t) = (M\Delta V/y)\left(1 - e^{-t/T_b}\right), \qquad 0 \le t \le T, \tag{5a}$$

$$= (M\Delta V/y)(e^y - 1)\, e^{-t/T_b}, \qquad T \le t; \tag{5b}$$

where

$$y = T/T_b. \tag{6}$$

Equations 5a and 5b evidently imply averaging over times short in comparison to T, such that the "fine structure" consisting of superposed individual pulses of risetime τ does not appear. (The detailed and average responses have been drawn schematically in Fig. 3.) The voltage pulse at the input of a pulse amplifier due to a group of M electronic charges traveling across, say, a gridded ionization chamber with transport time T, is also given by Eqs. 5a and 5b. In contrast to the CRM situation, however, the background here does not consist of individual charges traveling across the ion chamber at random times. Rather, the background is largely created only in the amplifier, where many electrons traverse a small space inside a vacuum tube or transistor with a much shorter transit time. The pulse amplifier thus requires a high-pass filter (differentiating element) of time constant T_a at the output to remove low-frequency noise generated in the circuit; the input time constant, T_b, is then made very much larger than T_a and can be neglected. In contrast, current pulse amplifiers,[12] used for example in connection with small fission chambers of a typical capacity $C_b = 20$ pF, deliver pulses across $R_b = 50$ ohms which makes the input time constant $T_b \approx 10$ nsec. In view of the orders-of-magnitude difference in relative parameters, the S/N ratio, although computed from equations of similar structure for the CRM and for the pulse amplifier, requires a different interpretation.



113-663

Fig. 3. Transient Response H(t) of Circuits Shown in Fig. 1. The input, with a transient of duration T superposed on a background at rate n, is shown in I; the response at the input of the CRM is shown in II. For easier visualization, rates and number of transient pulses are shown much smaller than in practical transient detection. For the pulse-amplifier circuit, the transient is a current pulse, but background is at a much higher rate; the response H(T) is, however, the same.

B. Signal-to-Noise Ratio; Single Time Constant

In considering now the S/N ratio, we must first define the "noise" generated by the background, as the root mean square of fluctuations in the CRM output level due to the superposition of the randomly occurring background. The mean-square fluctuation is equal to the second moment of the level distribution, which can be calculated through Campbell's theorem[13] giving the kth moment, λ_k:

$$\lambda_k = n \int_{-\infty}^{\infty} [F(t)]^k \, dt. \tag{7}$$

This yields, in connection with Eqs. 1a and 1b, a second moment,

$$\lambda_2 = \sigma^2 = n(\Delta V)^2 (T_b/x)\left(1 - \frac{1 - e^{-x}}{x}\right). \tag{8}$$

Given now that x is ordinarily of order 10^{-4} or smaller, Eq. 8 reduces to

$$\sigma^2 = n(\Delta V)^2 T_b/2. \tag{8'}$$

The derivation of Eq. 7, given for example in Ref. 7, presupposes that the number of superposed pulses, nT_b, is large. This theorem fails when $nT_b << 1$. In this case, the individual pulses are largely separated, and level excursions beyond ΔV are the result of occasional pileup only, such that a pileup approach, as suggested, for example, by Gold,[14] would be more appropriate. For the region of values of nT_b near unity, there appears to be no satisfactory theoretical treatment. This awkward circumstance limits the application of Eq. 7, as well as the equations developed in the following sections, to CRM monitoring situations where nT_b is relatively large, as will be reconsidered in detail further on.

Equation 8', in conjunction with the first moment of the distribution $\lambda_1 = nT_b \Delta V$, indicates that the relative fluctuations, $\sqrt{\lambda_2/\lambda_1}$, vary as $(T_b)^{-1/2}$. The switch selecting different values of C_b is therefore usually labeled "percent error" on general-purpose count-rate meters, which may appear to imply that a large value of T_b will result in a high S/N ratio. This is an erroneous judgment in transient detection, where the absolute (rather than the relative) magnitude of the fluctuations, which increases with $(T_b)^{+1/2}$, has to be considered, in conjunction with the effect of the time constant on signal amplitude.

Take the ratio of $H(T) = H_{max}$, the peak signal amplitude, to σ, from Eqs. 5a, 5b, and 8, and introduce the detectivity

$$D^2 = M^2/N, \tag{9}$$

where $N = nT$ = mean number of background pulses during the transient. One then finds the S^2/N ratio, R_0, at the CRM output

$$R_0^2 = H_{max}^2/\sigma^2 = (2D^2/y)(1 - e^{-y})^2 = D^2 U_0^2, \tag{10}$$

where U_0 = circuit-dependent part of the S^2/N ratio. Introducing now the function

$$h = (1 - e^{-y})^{-1}, \tag{11}$$

we may put

$$U_0 = \frac{\sqrt{2}}{h\sqrt{y}}. \tag{12}$$

This is plotted in Fig. 4 against $1/y$, i.e., against increasing T_b for fixed T. The maximum occurs at the point given by

$$e^y - 1 = 2y,$$

which comes to $y_{max} \approx 1.26R_{0_{max}}^2 \approx 0.81D^2$. The striking fact brought out by this plot is the optimization of transient detection for very small choices of the CRM time constant, that is, slightly smaller than the transient duration.

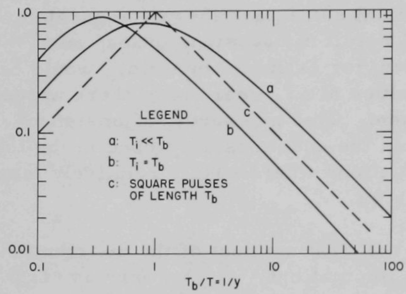


Fig. 4

$(S/N)^2 = R^2$, the Transient S/N Ratio at CRM Output, for Unit Detectivity $D^2 = M^2/N$. M = number of pulses in transient, N = mean number of background pulses during transient. Curve a: Without low-pass filtering ($T_i = 0$); Eq. 10. Curve b: With low-pass filter $T_i = T_b$; the square of the ratio R is plotted against $(T_b/T) = 1/y$. Curve c: Square pulses of length T_b.

113-653

C. Signal-to-Noise Ratio; Two Time Constants

The ratio R_0 can now be further improved, although only fractionally, by introducing a low-pass filter between the CRM output and the discriminator input. This decreases the peak response to the transient, but reduces the channel noise even more. The usual practice of displaying the CRM output by means of a servo-feedback recording potentiometer provides considerable low-pass filtering through the electromechanical inertia of the recorder. This hidden time constant, about 0.5 to 1 sec, can easily be too long where transients of shorter duration are expected. For optimum display, as considered earlier, one thus requires an oscillograph based on a low-inertia galvanometer, or a similar fast-acting display unit as described in Appendix E.

The influence of a low-pass filter, equivalent to an integrating time constant T_i, on the shape of individual pulses and transients is considered in detail in a number of treatises on the design of pulse amplifiers, for example, by Gillespie.[15] Introducing $z = T/T_i$, we find a transient response

$$H(t) = (M\Delta V/y)\,\frac{\left(1 - e^{-t/T_b}\right)\Big/y - \left(1 - e^{-t/T_i}\right)\Big/z}{1/y - 1/z}, \qquad 0 \le t \le T, \tag{13a}$$

$$= (M\Delta V/y\,\frac{(1/y)(e^{y} - 1)\,e^{-t/T_b} - (1/z)(e^{z} - 1)\,e^{-t/T_i}}{1/y - 1/z}, \qquad T \le t \le \infty, \tag{13b}$$

which comes to a peak value

$$H_{max} = (M\Delta V/y)(e^{y} - 1)^{z/(z-y)}(e^{z} - 1)^{y/(y-z)}, \tag{14}$$

always at or beyond T. The response to a single pulse, in view of $\tau << T$, is very closely approximated by the simpler equation,

$$F(t) = (\Delta V/y)\,\frac{e^{-t/T_b} - e^{-t/T_i}}{1/y - 1/z}, \qquad 0 \le t \le \infty. \tag{15}$$

Inserting this response in Eq. 7 and integrating, one finds the second moment,

$$\sigma^2 = [nT(\Delta V)^2/2y][z/(z+y)]; \tag{16}$$

hence the S^2/N ratio with integration becomes

$$R^2 = 2D^2(1/y + 1/z)(e^{y} - 1)^{2z/(z-y)}(e^{z} - 1)^{2y/(y-z)}. \tag{17}$$

Equation 17 is the analog of the signal-to-grid (base) current noise ratio, Eq. 26 of Ref. 15. As discussed in Ref. 15, as well as elsewhere, this equation is symmetric with respect to interchange of y and z, and turns out to peak for y = z. This choice of time constants thus always yields the best S^2/N ratio and will be provisionally adopted in the following discussions, although the S^2/N improvement, as may be judged from Fig. 4, is relatively modest. In Fig. 4, the ratio for y = z, which is

$$R^2 = \frac{4D^2}{h^2 y}\,\exp[-2y(h-1)] = D^2U_1^2, \tag{17'}$$

is plotted, as well as the ratio R_0^2 for $T_i = 0$, given by Eq. 10. The latter case is a reasonable approximation for any choice $T_i << T$. The peak for equal time constants is reached for

$$\sinh\,(y_{max}/2) = y_{max}/\sqrt{2},$$

or $y_{max} \approx 2.98$, and comes to $R \approx 0.88D^2$. For small values of y, $R^2 \approx 4D^2 y/e^2$. The S^2/N ratio given by Eq. 17' is also readily derived from the respective response functions for equal time constants,

$$F(t) = (\Delta V t/T_b) \exp(-t/T_b) \tag{18}$$

and

$$H(t) = MVe^{-t/T_b} e^y \left[\left(1 + \frac{t}{T_b}\right) \frac{1 - e^{-y}}{y} - 1\right]. \tag{19}$$

Equation 17' may again be simplified for purposes of computation. In terms of the function h(y) defined by Eq. 11, we may write

$$R^2 = D^2 U_1^2, \tag{17''}$$

and

$$U_1 = \left(\frac{2}{h\sqrt{y}}\right) \exp[-y(h-1)]. \tag{20}$$

Moreover, we are here not concerned with large values of the parameter y, for which the simple model transient assumption may have to be abandoned and a more specific transient shape introduced. For small values of y, an expansion, presented in detail in Appendix C, yields

$$U_1 = 2\frac{\sqrt{y}}{e}\left[1 - \frac{y^2}{24} + 1.1\left(\frac{y^2}{24}\right)^2 - 1.15\left(\frac{y^2}{24}\right)^3 + \ldots\right]. \tag{20'}$$

We shall use this expression in Section V. Equations 12 and 20, which are plotted in Fig. 4, point to a general policy of short-time constants $T_b \approx T$, whether T_i is chosen of comparable magnitude or longer.

At the same time, any "characteristic" signal that can be reasonably represented by piecing together sections of length T is also made adequately readable by this choice of CRM time constant, which thus appears to satisfy both quantitative and qualitative transient detection requirements. In contrast, input-circuit time constants for <u>pulse</u> amplification are usually chosen much longer than the duration of the current pulse T. This policy is partially due to the presence of white noise sources in the amplifier circuit, and partially the dependence of the maximum pulse amplitude H_m on T. The fractional shift, $\Delta H_m/H_m$, due to a fractional change in T, $\Delta T/T$, comes to

$$\frac{\Delta H_m}{H_m} = -\frac{\Delta T}{T}\,\frac{1 - y^2 e^y}{(e^y - 1)^2}$$

$$= -\frac{\Delta T}{T}\left(\frac{y^2}{12} - \frac{y^4}{240} + \ldots\right)$$

for $T_i = T_b$. The basic uncertainty ΔT for a gridded ion chamber, for example, arises from the finite length and random orientation of ionization tracks. Thus the current pulse, formed as electrons from different parts of the track emerge through the grid and pass to the anode, actually deviates markedly from the square shape it would have for a strongly localized charge. A similar situation exists in many transient monitors; here, the distribution of sources plays the part of the ion track, and a transient of predictable shape occurs only for a point source moving past the detector. Thus, the precision of pulse-height information, which is of fundamental importance for the ion chamber or similar detector, is emphasized by deliberately making the output signal insensitive to input pulse shape; in contrast, shape is the principal information content of the signal for transient detection.

With this strong emphasis on short time constants T_b, one should not, however, lose sight of the restriction of the foregoing S/N ratios to large values of nT_b. This restriction implies that a given transient detector with very low background cannot be described meaningfully by Eqs. 10 or 17'. However, for such a case, a relatively more readable record is still obtained with a short time constant than with a long one. Finally, Eq. 17' shows the considerable benefit obtainable from the use of additional detectors (or more sensitive detectors) in a given transient monitor, through the dependence of readability on $M^2/N = D^2$.

D. Final Clipping Stage

Thus far, we have treated a filter with two time constants. When the processor output is to be presented to an alarm discriminator, the verdict of the latter must be unaffected by possible slow fluctuations in the background rate, which up to now has been treated as constant. Actually, the background is constant only in special transient-detection situations. In a more typical application, the detection of fuel failures discussed in Section II.B, the reactor plant may be operated for some time with a number of small fuel-cladding leaks, if the danger of further damage does not warrant shutdown, location, and replacement of the leaking fuel elements. Moreover, a certain amount of background may be due to activation of the coolant; hence the background level will reflect the power level with a delay characteristic of the decay constant of the activity. This suggests that, rather than referring the alarm-discriminator signal input to a fixed dc level, a reference level be used that is the average of the CRM signal over a time sufficiently long in comparison to its transient response. Alternatively, dc components may be subtracted from the signal input by introducing a suitably dimensioned blocking capacitor. This mode of operation makes the alarm discriminator still more insensitive to small leaks, which are already inefficiently detected by a processing channel whose time constants T_b are adjusted to maximize the S^2/N ratio for short transients. As mentioned in Section II, another processing channel may be fed by the same detectors to meet the requirement of leak detection as well as the more important requirement of fast transient detection. The leak signal processor then calls for long time constants throughout. More generally, when several transients of different duration are encountered in transient monitoring, an equivalent number of optimized processing channels are required for efficient detection. The use of a final clipper may be regarded as a typical feature of monitoring systems in which transients exceeding a certain level trip an alarm, in contrast to ordinary count-rate metering of nuclear radiation.

The final clipping stage (shown in Fig. 1a for the transient monitor channel) is evidently quite analogous to the clipping stage of a pulse amplifier (shown in Fig. 1b). For the latter case, the input time constant T_b is usually kept very long in comparison with T_a, which thus determines the lower-frequency cutoff. In contrast, the final clipping time constant is always longer than T_b for the transient monitor.

The effect of a linear filter with three arbitrary time constants is considerably more complex than the foregoing cases of one or two time constants. We therefore present detailed calculations in Appendix B, from which we quote here only the S^2/N ratio for three equal time constants. Introducing the function

$$Z^2 = 1 + y^2h(h - 1), \tag{21}$$

we obtain

$$R^2 = D^2U_2, \tag{22}$$

and

$$U_2 = 2U_1(Z - 1)\exp(Z - 1). \tag{23}$$

Equation 23 is plotted in Fig. B.1, which shows that a short final clipping stage does not improve the S^2/N ratio, but may be useful for other reasons.

In the practical example of a fuel-failure monitor with alarm, considered above, it is expedient to keep T_a sufficiently short to take out fluctuations of, say, 10 min or more, which should readily remove variations in background level due to activation processes. A value of T_a in this range then still makes the inequality $T_a >> T_b$ valid; hence the channel response is closely approximated by the two-time-constant filter response, U_1.

E. Digital Count-rate Meter

In concluding this section, we may add a brief remark on the response obtainable with a digital processor, i.e., a unit that processes the input directly in such a way that the content of a digital store represents the input rate. "Digital" count-rate meters are offered by several manufacturers; such units, however, merely accumulate input counts for a fixed interval, display the count, reset, and restart. The memory characteristic of analog count-rate meters can be provided by subtracting a fixed fraction of the content of a scaler continuously fed by the input at certain intervals. Alternatively, a fixed number, corresponding to a certain fraction of the mean content, may be subtracted. These types of digital units simulate the leakage of charge from the bucket capacitor, and thus are describable in terms of the equations developed above to the extent to which this simulation succeeds.

To effect the subtraction requires elaborate circuitry. A more straightforward digital count-rate meter can be implemented by accumulating input pulses in an add-subtract scaler and feeding them into a delay T_b, after which each pulse is subtracted. When such an instrument is turned on with a clear scaler at a steady input rate n, the scaler contents will increase linearly for T_b, and then continue to fluctuate about nT_b, just as would an analog circuit in which randomly arriving square pulses of length T_b are added linearly. Delays of several seconds can be effected through a large shift register,[16] driven by a clock oscillator at a frequency f such that $fT_b = m$ = number of bits in the shift register. To avoid deadtime loss of counts, we require that $f \gg n$. The process is then governed by Bernoulli (binomial) statistics; the mean store content, nT_b, fluctuates with a variance $nT_b(1 - nT_b/m)$. Neglecting the small term $nT_b/m = n/f$, one readily finds the S/N ratios

$$R_d^2 = \left.\begin{aligned} &D^2/y, \quad y \geq 1, \\ &D^2 y, \quad y \leq 1, \end{aligned}\right\} \qquad (24)$$

with a maximum value slightly larger than for the analog case. Figure 4 indicates, however, that this better S^2/N ratio is maintained only over a narrow range of the parameter y. For those types of transient monitors in which the duration of the signal can be determined within certain limits (e.g., from the sweep speed), a digital processor of the type described here may offer a slight advantage as regards S^2/N ratio, in addition to the more significant advantage of stability.

For other types of transient monitors, in which the duration of the transient cannot be precisely estimated, the S^2/N ratio of the digital CRM is still comparable to that obtaining for a corresponding analog device. For any system intended for continuous surveillance, stability is a crucially important factor, which may make the relatively high initial cost of digital equipment worthwhile. The digital unit is directly amenable to several options in readout or storage of information which have become available in connection with digital computing equipment. Concerning the application of a digital CRM to fuel-failure monitoring or similar tasks, note that the unit described above is affected by background variations. Thus the alarm cannot be actuated simply from a certain scaling stage without changing the detection efficiency as a function of background level, as discussed above. The alarm scaling stage may, however, be gated from a second scaler in which background is accumulated for a fixed time, whereupon the scaler is reset and restarted. Alternatively, the pulses emerging from the shift register may be subtracted twice and fed to another shift register, whose output is added. This "DD2" *modus operandi* allows a direct setting of the alarm. Other advantages resulting from the symmetry of the "response" described by this kind of system are discussed in Appendix B.

IV. PERFORMANCE OF THE ALARM DISCRIMINATOR AT HIGH BACKGROUND RATES

The development up to this point mainly concerned the quality of the presentation of a transient, which involved maximizing the S^2/N ratio. This parameter peaks for time constants comparable to the transient duration, whether or not the bandwidth was restricted by an "integrating" filter. We turn now to a discussion of the quality of performance of the alarm discriminator, which can best be described through two statistical quantities, the detection probability and the false-alarm frequency.* To make these quantities as independent of fluctuating background and other uncontrollable factors as possible, dc components must be subtracted, as described in Section III. This type of operation of an alarm discriminator may be characterized as MNR (Mean Noise Reference level) in contrast to DCR (DC Reference level). For quantitative evaluation, two essential assumptions in addition to the model transient shape must now be introduced. The intensity of a "typical" transient must be specified, and this intensity must be assumed to be well above the noise sampled during the excursion.

These model assumptions are perhaps more realistic for some applications of transient monitoring than for others. In particular, the transients one can reasonably expect in fuel-failure monitoring vary considerably in intensity as well as shape, depending on the type of cladding burst, degree of dispersion, and flow pattern of the plant, as discussed elsewhere.[17] In the sense used here, a "typical" signal thus implies the sudden release of, say, one-third of the fission products accumulated in the bonding sodium of a fuel pin, with ideally turbulent flow (hence, predictable decay en route) dispersed through perhaps 10 liters during transit and proportionately sampled by the monitor loop. More violent cladding failures should result in correspondingly more intense signals. In a number of other transient monitors, one may similarly define a minimum transient strength, or perhaps a range of transient strengths, which still allows a quantitative estimate of the detection probability for each at a given alarm level and processor filter response. In the discussion here, only the simplest model will be used to derive the detection probability; further on, the false-alarm frequency will be considered, for which a more extensive mathematical framework is necessary.

A. Detection Efficiency

To obtain an expression for the detection efficiency, suppose that the alarm discriminator is tripped by a transient containing A or more pulses, arriving within T at a quasi-uniform rate. The probability that

The term "detection probability" is used here in preference to the more common "detection efficiency" to stress the statistical nature of this parameter and distinguish it from the purely instrumental efficiency of the nuclear detectors. "False-alarm frequency" likewise implies a probability per unit time. (It is perhaps unfortunate that "frequency" denotes two rather different concepts, rendered in German, for example, a "typical" transient contains a certain number of pulses is assumed to be given by a Poisson distribution. Such would be the case for an approximately constant release of fission products, approximately evenly transported past the detectors, in a fuel-failure monitor. The mean number of pulses in the typical transient, M, is thus also the variance. With a mean background rate, n, the background sample during the transient comes to $nT = N$, which is subtracted from the latter, but still influences the size of the total variance, increasing it to $M + N$. In the limit of large numbers of pulses, the Poisson distribution may be conveniently approximated by a Gaussian distribution, which allows one to define the detection probability $g(G')$ directly in terms of tabulated functions:

$$g = \tfrac{1}{2}[1 + \Phi(G')], \tag{25}$$

where

$$G' = \frac{M - A}{\sqrt{2(M+N)}}, \tag{26}$$

and

$$\Phi(G') = \frac{2}{\sqrt{\pi}} \int_0^{G'} e^{-t^2}\, dt$$

is the well-known error integral.[18] Now let

$$G = \frac{M - A}{\sqrt{2M}}. \tag{26'}$$

Then, for $N/M << 1$, as assumed explicitly here, we may express G' in terms of G by expanding the root and further expand Eq. 25 in a Taylor series, yielding the approximation

$$g = \tfrac{1}{2}[1 + \Phi(G)] - \frac{GN}{2M\sqrt{\pi}} e^{-G^2} + \dots . \tag{25'}$$

Practical considerations demand detection probabilities not smaller than, say, 0.9, for which the leading correction term in Eq. 25' comes to approximately $0.1\,N/M$. Given $N/M \leq 0.1$, the whole correction can be safely neglected, and the detection probability is thus effectively independent of slow excursions in the background rate.

An upper limit is imposed on the detection probability through the necessary tradeoff of detection probability against false-alarm frequency;

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*Assuming the subtracted background is effectively measured over a time interval much longer than T.

such a limit might be reasonably set at 0.99. For $0.90 \leq g \leq 0.99$, Eq. 25' can be conveniently approximated by a simpler expression,

$$g = 1 - Be^{-pG}, \qquad (25'')$$

which is more accurate over this range than the error integral expansion for large values of the argument given in standard reference works on error integrals.[18] The error involved in using Eq. 25" rather than Eq. 25' may be judged from Fig. 5 in which both expressions are plotted; the constants $B = 1.9$ and $p = 3.17$ are used here to give the best fit. Figure 6 is a conventional plot of g versus G on "probability" paper.

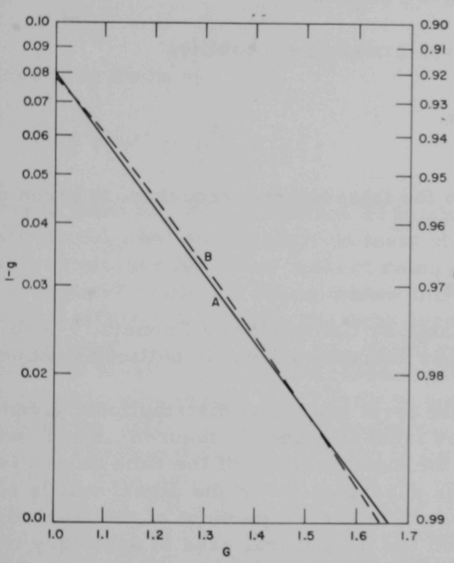

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Fig. 5. Detection Probability g against Parameter G Defined by Eq. 26'. Curve A: Approximation over the range of interest, Eq. 25". Curve B: Eq. 25.

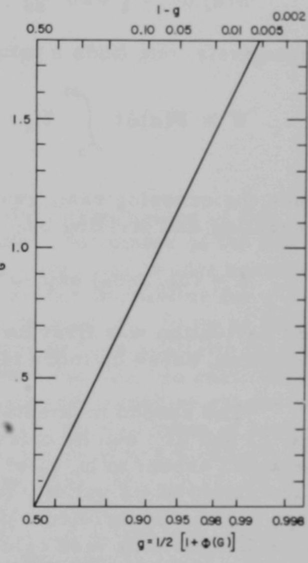

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Fig. 6. Plot of g versus G on Probability Paper

B. False-alarm Frequency

We shall now derive the false-alarm frequency, first in an elementary treatment and then in a more thorough fashion. The elementary treatment is analogous to the well-known problem of electronic noise, with the important proviso that this is applicable only in the limit of high background rates and/or long time constants, i.e., where nT_b is a very large number.

In that limit, it is plausible to assume a Gaussian distribution of the analog level about its mean, zero, as well as a corresponding Gaussian distribution of the time derivative of the analog signal. A level crossing is then assumed to occur whenever the signal is within some small distance da below the level a and at the same time has a positive slope s. The time Δt required by this is determined by the condition $da = s\Delta t$.

With the level and slope distributions

$$P(a)\,da = \left(\sigma_a \sqrt{2\pi}\right)^{-1} \exp(-a^2/2\sigma_a^2)\,da, \tag{27}$$

and

$$P(s)\,ds = \left(\sqrt{2\pi}\,\sigma_s\right)^{-1} \exp(-s^2/2\sigma_s^2)\,ds, \tag{27'}$$

respectively, one finds a total level-crossing probability

$$W = P(a)\Delta t \int_0^\infty P_s \cdot s\,ds. \tag{28}$$

Hence the crossing rate, equal to the false-alarm frequency, is given by integrating and dividing by Δt to yield

$$f = (\sigma_s/2\pi\sigma_a)\exp(-a^2/2\sigma_a^2). \tag{29}$$

This derivation was first formulated by Campbell and Francis.[19] A different derivation, based on more rigorous considerations, is outlined in Appendix A.

The second moments of the level and slope distributions, given in Eqs. 27 and 27', can be calculated from Campbell's theorem, Eq. 7, which one would expect to be valid for the superposition of the time derivatives to the extent of its validity for the superposition of the direct values of a large number of individual pulses. The second moment of the level distribution has already been calculated for the general case of arbitrary time constants (cf. Eq. 16). For the choice $T_i = T_b$,

$$\sigma_a = \Delta V \sqrt{nT_b}/2. \tag{30}$$

Similarly, σ_s is obtained by differentiating Eq. 18 with respect to time, inserting this "slope response" into Eq. 7, and integrating, with the result

$$\sigma_s = (\Delta V/2)\sqrt{n/T_b}. \tag{31}$$

The level a in Eq. 28 must now be connected with the number of pulses A in the transient that will just trip the alarm level when the latter is set for a detection probability g (cf. Eqs. 25 and 25"). Evidently the level is just equal to the maximum of the response to a transient of A (rather than M) pulses, given by Eq. 14 if A is substituted for M. Combining

Eqs. 14, 26', 29, 30, and 31, one thus finds the false-alarm frequency for equal time constants $T_i = T_b$, when an MNR-connected alarm discriminator is set at a detection probability g for transients of expected duration T, containing a mean number of pulses M, in the presence of a background of mean rate n:

$$\log f = \log y - \log 2\pi T - \tfrac{1}{2} D^2 U_1^2 W^2(g), \tag{32}$$

where U_1, the filter response for two equal time constants, is given by Eq. 20. The function W(g) depends on the required detection probability; hence again on mean transient pulse number M,

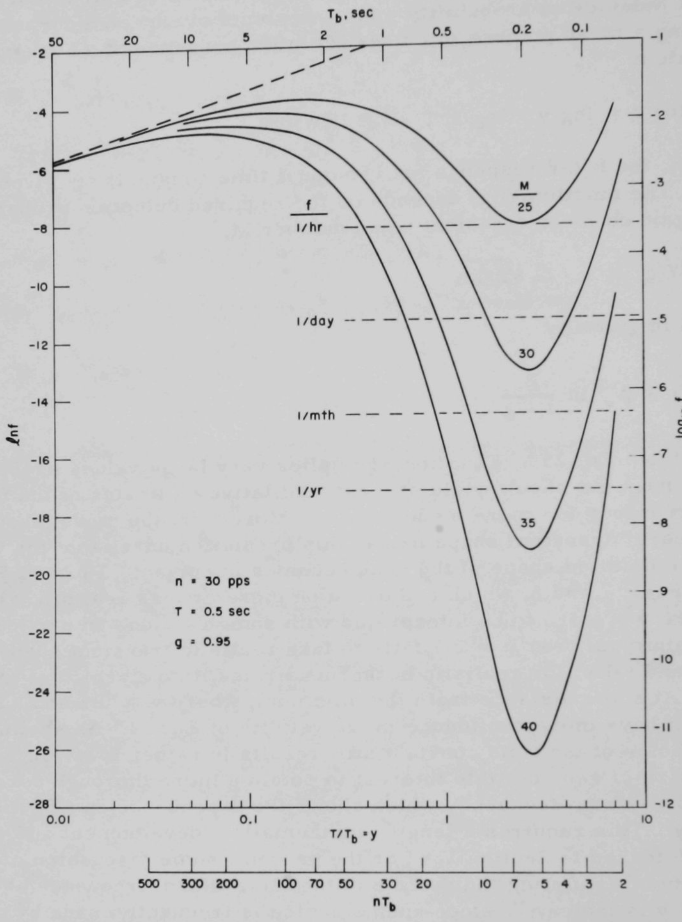
$$W(g) = 1 - G\sqrt{2/M}, \tag{33}$$

where G is given by

$$G = p^{-1} \ln \frac{B}{1 - g},$$

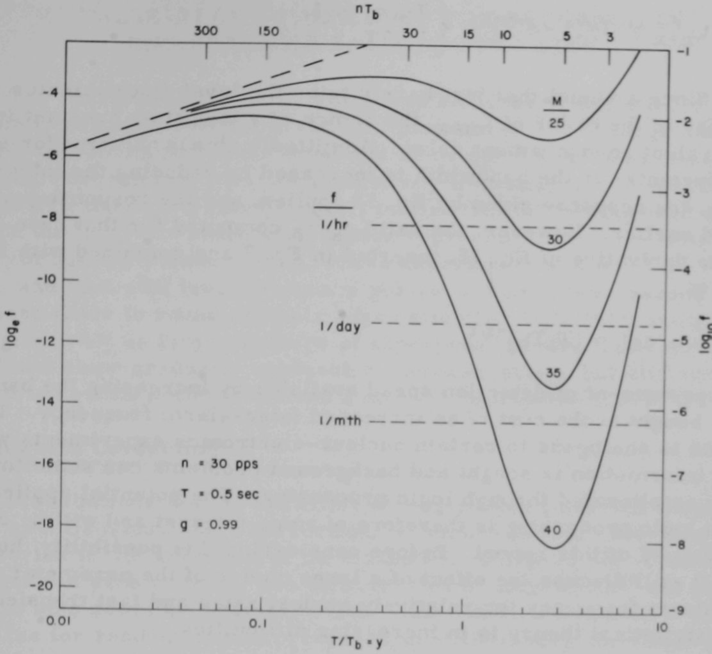
derived from Eq. 25". Equation 32 implies very large values of the statistic nT_b, but should yield at least qualitative estimates of the false-alarm frequency for more modest rates. Moreover, the model assumption of a "square" transient shape makes this prediction unreliable for $y \approx 1$, where the detailed shape of the peak becomes important. Plots of Eq. 32, such as Figs. 7 and 8, which are based on more or less realistic background rates, are therefore to be interpreted with some caution. In particular, the strong minimum year $y = 2.7$ fails to take realistic transient shapes into account and falls, for realistic background rates, into a region of low values of nT_b. At some distance from the minimum, where $y \approx 0.5$ and $nT_b \approx 30$, one might have more confidence in the validity of Eq. 32. As shown in Fig. 7, such a choice of the time constant still results in rather low values of f. It is therefore of considerable interest to obtain a more thorough derivation of the false-alarm frequency which shows the dependence on nT_b more explicitly. This requires a lengthy mathematical development and will thus be deferred to Section V. For the present, some discussion of the possibility of exploiting the decline of the false-alarm frequency at small values of y is indicated, since such a choice is frequently made by operating personnel who have been using count-rate meters for monitoring nontransitory signals.

Small values of y result in small values of the last term of Eq. 32. This term is responsible for the strong minimum, such that the first term, log y, dominates. Now $W^2(g)$ and D^2 are independent of y; hence, small values of the last term imply a very low response $U_1(y)$. This leads to the apparently paradoxical result that the false-alarm frequency declines as the signal peak becomes smaller than the rms background. However, although purely statistical background fluctuations under these conditions



113-657

Fig. 7. False-alarm Frequency at Fixed Background Rate n = 30 pps and Transient Duration T = 0.5 sec, for Various Values of M (number of pulses in the transient), as a Function of the Time-constant Parameter T/T_b (T_b = integration and differentiation time), with Detection Probability g = 0.95 (Eq. 32)



113-658

Fig. 8. False-alarm Frequency at Fixed Background Rate n = 30 pps and Transient Duration T = 0.5 sec, for Various Values of M (number of pulses in the transient), as a Function of the Time-constant Parameter T/T_b (T_b = integration and differentiation time), with Detection Probability g = 0.99 (Eq. 32)

are, on the average, higher than the response to the transient, they are also considerably reduced in frequency. Hence the rate of crossing the alarm level may still be low. As y becomes very small, the false-alarm frequency becomes virtually independent of the detection probability and approaches the frequency of crossing the mean level in the positive direction or "zero-crossing" frequency. This may appear to suggest that small choices of the time-constant parameter y allow independent optimization of both transient-detection quality parameters. Such a conclusion is, however, largely illusory for two practical reasons: First, all variations of the background rate other than purely statistical are ignored in the above argument; second, equipment stability is not taken into account. Settings of the alarm discriminator close to the mean level obviously change drastically as the mean level shifts, and similarly are difficult to maintain with the required degree of stability over long periods. Moreover, we have thus far not considered the third transient-detection quality parameter, the announcement delay, which is strongly dependent on the choice of y. The response H(t,T) for equal time constants, given by Eq. 19, peaks at a time

$$t_{max} = T/(1 - e^{-y}) = \begin{cases} T_b, & y \ll 1, \\ 1.05T, & y = 3. \end{cases} \tag{34}$$

Since a signal that just barely trips the level discriminator results in a delay of the order of t_{max}, the choice of a long time constant implies an equivalent announcement delay. Admittedly, this is only true for equal time constants; if the bandwidth is increased by reducing the integrating time T_i, the response given by Eq. 12 applies, and the response peak is reached earlier. However, the ratio σ_s/σ_a computed for that case from the time derivative of Eq. 15, inserted in Eq. 7 and combined with Eq. 16, comes to

$$(\sigma_s/\sigma_a) = (T_i T_b)^{-1/2}. \tag{35}$$

The improvement of detection speed available by increasing the bandwidth is thus bought at the cost of an increased false-alarm frequency. This situation is analogous to certain nuclear-electronics experiments where timing information is sought and background problems can sometimes be greatly ameliorated through logic processing. The potential application of such logic processing is therefore of some interest and will be discussed in Section VI of this report. Before considering this possibility, however, we must still discuss the effect of a large choice of the parameter y on the false-alarm frequency for relatively modest rates and fast transients, where statistical theory is in increasing difficulties.

V. FALSE-ALARM FREQUENCY FOR PRACTICALLY ENCOUNTERED BACKGROUND RATES

The derivation of Eq. 31, which was sketched in Section IV, specifically assumes that the statistical parameter nT_b is large enough to allow the approximation of the level and slope distributions by Gaussians, even though the random superposition of any finite number of specific functions F(t) must necessarily result in distributions that depend to some extent on the level and slope distribution represented by F(t). This is evident for ideally square pulses, whose superposition is a "staircase" pattern, and thus still true for square pulses that have been passed through a low-pass filter to round off their edges slightly. With increasingly heavy filtering, as well as large numbers of superposed pulses, the distributions of level and slope gradually approach a Gaussian shape, but still must be expected to deviate noticeably from a normal distribution far in the "wings."

A. Edgeworth Correction

Distributions that do not differ strongly from Gaussian shape can be written as a Gaussian and a correction, which can in turn be developed in a power series in the argument of the Gaussian. The problem of finding the coefficients of such a series was first treated by Edgeworth[20] and is discussed in less detail by Cramer.[21] For a distribution whose first moment is zero (as for random counts processed in MNR-mode), the Gaussian parameter comes to

$$\beta = a/\sigma_a, \tag{36}$$

where a = level above zero, and $\sigma_a = \sqrt{\lambda_2}$ = second moment of the trace. The distribution, in terms of this parameter, is given by[20,21]

$$P(\beta) = \phi(\beta) - A\frac{d^3\phi}{d\beta^3} + \left(B\frac{d^4\phi}{d\beta^4} + \frac{A^2}{2}\frac{d^6\phi}{d\beta^6}\right) - \ldots, \tag{37}$$

where

$$A = (1/3!)(\lambda_3/\lambda_2^{3/2}), \tag{38a}$$

$$B = (1/4!)(\lambda_4/\lambda_2^2), \tag{38b}$$

and

$$\phi = (2\pi)^{-1/2}\exp(-\beta^2/2).$$

The moments of the distribution in coefficients A and B evidently depend on the filter, through Campbell's theorem. The derivatives of the Gaussian are

$$\frac{d^3\phi}{d\beta^3} = -(\beta^3 - 3\beta)\,\phi,$$

$$\frac{d^4\phi}{d\beta^4} = (\beta^4 - 6\beta^2 + 3)\,\phi,$$

and

$$\frac{d^6\phi}{d\beta^6} = (\beta^6 - 15\beta^4 + 45\beta^2 - 15)\,\phi.$$

Evaluating the moments and coefficients for a filter with two time constants from the respective Campbell integrals (Eq. 7), one obtains the values shown in Tables I and II. For equal time constants, which will be assumed henceforth, successive terms in the Edgeworth series are proportional to half-powers of $(nT_b)^{-1}$ and thus decline rapidly when n is large. For that reason, the Gaussian approximation is entirely adequate for problems involving electronic noise. For the present case, in contrast, the series may well fail to converge beyond a certain value of the parameter β, given a relatively small input rate n and a small choice of the filter constant T_b. The validity of Campbell's theorem (used for evaluating the moments) is also questionable for small values of nT_b. Therefore the convergence of the series may be taken as an indication of the limit of validity of any treatment of the trace distribution based on statistical considerations. A somewhat more convenient equivalent criterion will be discussed further on in this section.

TABLE I. Semi-invariants λ_k of the Trace Distribution Generated by a Random Count Rate n in a CRM with Time Constant T_b and Filter Time Constant T_i

Moment Order, k	(a) T_i and T_b Arbitrary	(b) $T_i = T_b$	(c) $T_i \ll T_b$
1	nT_b	nT_b	nT_b
2	$\frac{T_b}{T_b + T_i}\,\frac{nT_b}{2}$	$\frac{nT_b}{4}$	$\frac{nT_b}{2}$
3	$\frac{T_b^2}{(T_b + T_i)^2 + \frac{1}{2}T_bT_i}\,\frac{nT_b}{3}$	$\frac{2}{9}\,\frac{nT_b}{3}$	$\frac{nT_b}{3}$
4	$\frac{T_b^3}{(T_b + T_i)\left[(T_b + T_i)^2 + \frac{4}{3}T_bT_i\right]}\,\frac{nT_b}{4}$	$\frac{3}{32}\,\frac{nT_b}{4}$	$\frac{nT_b}{4}$
k	-	$\frac{(k-1)!}{k^{k-1}}\,\frac{nT_b}{k}$	$\frac{nT_b}{k}$

TABLE II. Edgeworth Series Coefficients for the Same Filter

Coefficient	T_i and T_b Arbitrary	$T_i = T_b$	$T_i \ll T_b$
$\frac{1}{3!}\frac{\lambda_3}{\lambda_2^{3/2}}$	$\frac{\sqrt{2}}{9}\frac{\sqrt{T_b(T_b+T_i)}}{(T_b+T_i)+T_bT_i/2(T_b+T_i)}(nT_b)^{-1/2}$	$\frac{8}{81}(nT_b)^{-1/2}$	$\frac{\sqrt{2}}{9}(nT_b)^{-1/2}$
$\frac{1}{4!}\frac{\lambda_4}{\lambda_2^2}$	$\frac{1}{24}\frac{T_b(T_b+T_i)}{(T_b+T_i)^2+\frac{4}{3}T_bT_i}(nT_b)^{-1}$	$\frac{1}{64}(nT_b)^{-1}$	$\frac{1}{24}(nT_b)^{-1}$
$\frac{1}{2}\left(\frac{1}{3!}\frac{\lambda_3}{\lambda_2^{3/2}}\right)^2$	$\frac{1}{81}\frac{T_b(T_b+T_i)}{[(T_b+T_i)+T_bT_i/2(T_b+T_i)]^2}(nT_b)^{-1}$	$\frac{32}{81^2}(nT_b)^{-1}$	$\frac{1}{81}(nT_b)^{-1}$
$\frac{1}{k!}\frac{\lambda_k}{\lambda_2^{k/2}}$		$\frac{1}{k}\left(\frac{2}{k}\right)^k(nT_b)^{1-(k/2)}$	$\frac{2^{k/2}(nT_b)^{1-(k/2)}}{k\cdot k!}$

To find the false-alarm frequency based on a quasi-Gaussian level distribution, we write an equivalent distribution for the slope,

$$P(\gamma) = \phi(\gamma) - A'\frac{d^3\phi}{d\gamma^3} + \left(B'\frac{d^4\phi}{d\gamma^4} + \frac{A^2}{2}\frac{d^6\phi}{d\gamma^6}\right) - \ldots, \tag{39}$$

whose coefficients are similarly derived from Campbell integrals involving the single-pulse response dF/dt. For the foregoing case of equal time constants, $dF/dt = (\Delta V/T_b)(1 - t/T_b)\exp(-t/T_b)$, from which one readily finds

$$A' = \frac{16}{81}\frac{1}{\sqrt{nT_b}} \tag{39a}$$

and

$$B' = \frac{5}{64}\frac{1}{nT_b}. \tag{39b}$$

The false-alarm frequency is now calculated as in Section IV by integrating the joint level-slope distribution over all positive slopes and dividing by Δt to give

$$f = \left(\frac{\lambda_2'}{4\pi^2\lambda_2}\right)^{1/2} e^{-\beta^2/2}[1 + A(\beta^3 - 3\beta) + \ldots]\int_0^\infty \gamma\, d\gamma\, P(\gamma). \tag{40}$$

The integration is straightforward. Using the identity

$$\int_0^\infty \gamma\frac{d^k\phi}{d\gamma^k}d\gamma = \left.\frac{d^{k-2}\phi}{d\gamma^{k-2}}\right]_0, \tag{41}$$

one finds that the first-order (odd-power) correction term vanishes, and the integral comes to

$$\int_0^\infty \gamma \, d\gamma \, P(\gamma) = (2\pi)^{-1/2}\left[1 - \left(B' - \frac{3}{2}A'^2\right)\right] = (2\pi)^{-1/2}\left(1 - \frac{0.0196}{nT_b}\right). \quad (42)$$

The next higher correction term, omitted here, is of odd power in the parameter γ and therefore also vanishes.

The expression thus obtained for the false-alarm frequency still is cast in a form that contains its dependence on the filter time constant T_b both directly and indirectly through β. The latter parameter can be expressed in convenient form by an expansion (presented in Appendix C),

$$\beta = W(M;g)\,R(D;y) = WDU_1$$

where, according to Eq. 20',

$$U_1 = (2\sqrt{y}/e)(1 - y^2/24 + 1.1\,y^4/576 - \ldots), \quad (43)$$

which is a fair approximation up to $y \approx 1$. Moreover, by introducing the variable

$$u = y/N = 1/nT_b, \quad (44)$$

where, as before, $N = nT$, we can separate β into a part that depends (at least to a first approximation) only on the transient strength M and specified detection probability g, and a simple function of u:

$$\beta \approx H\,\sqrt{u}(1 - \epsilon^2u^2 + 1.1\,\epsilon^4u^4 - \ldots), \quad (45)$$

where

$$H = (2M/e)[1 - (2/M)^{1/2}G] \quad (46)$$

and

$$\epsilon^2 = N^2/24. \quad (47)$$

Collecting all terms, we obtain the false-alarm frequency as a function of the variable u and constants H and ϵ^2 only:

$$\log(f/n) = \log(u/2\pi) + \log[1 - E(u)] - \frac{H^2u}{2}(1 - 2\,\epsilon^2u^2 + 2.11\,\epsilon^4u^4 - \ldots), \quad (48)$$

where

$$E(u) = k_1u - k_2u^2 + k_3u^3 - k_4u^4 + k_5u^5 - \ldots, \qquad (49a)$$

with coefficients

$$\left.\begin{aligned} k_1 &= 3(Ha + b) + c, \\ k_2 &= H^3a + H^2d + 3(Ha + b)\,c, \\ k_3 &= H^4e + 3Ha\epsilon^2 + (H^3a + H^2d)\,c, \\ k_4 &= H^6a^2/2 - 6H^2d\epsilon^2 - 3H^3a\epsilon^2 + (H^4e + 3Ha\epsilon^2)\,c, \\ k_5 &= \epsilon^2[4H^4e - 3.3\,Ha\epsilon^2] + H^6a^2/2 - (6H^2d\epsilon^2 - 3H^3a\epsilon^2)\,c, \\ &\vdots \end{aligned}\right\} \qquad (49b)$$

The constants

$$\left.\begin{aligned} a &= 0.0987654, \\ b &= 0.0087615, \\ c &= 0.0195973, \\ d &= 0.1257285, \\ e &= 0.0575345, \\ &\vdots \end{aligned}\right\} \qquad (49c)$$

are derived from the coefficients given in Table II, specifically for $T_i = T_b$.

B. Criterion of Validity Limit

The form of Eq. 48 suggests the adoption of a validity criterion based on the correction term $E(u)$. Given a certain set of values $\{H,\epsilon\}$ and a filter whose integrating and differentiating time constants are adjustable concurrently, the false-alarm frequency estimate provided by Eq. 48 becomes increasingly unreliable as $E(u;H,\epsilon)$ approaches unity. Equation 48, in particular, and statistical theory, in general, become inapplicable for values of the parameter u for which $E(u)$ becomes so large that Eq. 49a fails to converge.

To illustrate this limit, we shall use a numerical example based on the parameter values contained in Figs. 7 and 8 ($M = 25, 30, 35, 40$; $g = 0.95, 0.99$). The values assumed by the parameter H are given in Table III and plotted in Fig. 9. With $N = 15$ ($n = 30$, $T = 0.5$ sec) as specified for Figs. 7 and 8, Eq. 48 yields the false-alarm frequencies plotted in Fig. 10, for $H = 10, 20$, and 30, respectively. Values for an "uncorrected" Gaussian distribution, according to Eq. 32, are also plotted for comparison.

TABLE III. Values of the Parameter $H = (2M/e)(1 - G\sqrt{2/M})$ for Certain Choices of M and g

M	H: g = 0.95	H: g = 0.99
25	12.66	9.83
30	15.43	12.70
35	18.56	15.65
40	27.80	18.61

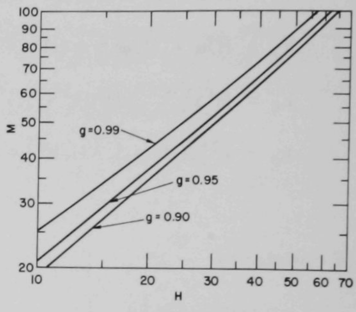


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Fig. 9. The Parameter H versus M, for Certain Choices of Detection Probability g

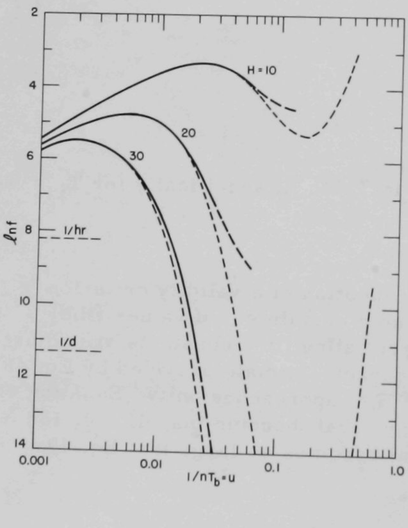

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Fig. 10

False-alarm Frequency for Parameters $n = 30$, $T = 0.5$ sec as a Function of $u = (nT_b)^{-1}$, for Different Choices of $H(M;g)$, According to Eq. 48 (solid lines) and Eq. 32 (broken lines). Equation 48, an improved version of Eq. 32, becomes statistically unreliable where the heavy lines are broken and may be inferred to fail entirely where these lines stop.

Figure 10 demonstrates that the spectacular reduction of the false-alarm frequency predicted by Eq. 32 does not occur when we consider a more realistic distribution, which differs only negligibly from a Gaussian in the peak region, but is much stronger for large values of the parameter. Sample values of the correction E, given in Table IV, show that this correction becomes negative in the peak region, as one would expect, to compensate for the positive correction outside. Considering, as discussed above, that Eq. 48 is not valid when the correction E becomes much larger than unity, the false-alarm frequency according to Eq. 48 is drawn as a solid line up to that point, and continued a small distance beyond to show the trend. Predictions according to Eq. 32 are shown as a broken line. Thus, for example, a false-alarm frequency of less than once per day for H = 20 would appear to be achievable if one were to rely, wrongly, on Eq. 32. Equation 48 makes it apparent that a much higher false-alarm frequency is likely to prevail. To the extent to which the parameters chosen for the above numerical illustration are realistic, it points to the need for further improvements; some possibilities in this regard are discussed in Section VI.

TABLE IV. Corrections E(u) for H = 10, H = 20, and H = 30, According to Eq. 48

H = 10		H = 20		H = 30	
u	E(u)	u	E(u)	u	E(u)
0.1	+1.243	0.05	+4.1	0.033	+5.65
0.06	+0.218	0.03	+0.781	0.020	+1.251
				0.0133	+0.478
0.04	+0.0467	0.02	+0.233	0.0066	+0.0562
0.01	-0.0194	0.005	-0.009	0.00033	-0.0007
0.001	-0.003	0.001	-	-	-

C. Optimum Choice of Parameters

With this improved understanding, we now consider the practical matter of finding the best choice of time constants and alarm-level settings for a transient detection system of known parameters M, T, and n.

The first question of interest in that connection is the best tradeoff between false-alarm frequency and detection probability, assuming that the performance parameters are not good enough to satisfy an initial specification of, say, less than 1% chance of missing the transient at false-alarm frequency no higher than once a week of continuous operation (1.6×10^{-6}). We may further suppose that a 5% chance of missing is still barely acceptable and a somewhat higher false-alarm frequency might be countenanced. The following useful relation between these two performance parameters is obtained by differentiating Eq. 32 with respect to the detection probability:

$$\frac{d \log f}{d \log g} = \frac{WgR^2}{p(1-g)}\sqrt{\frac{2}{M}} . \tag{50}$$

In practice, the number of pulses in the transient, M, will lie between 10 and 100. (For smaller numbers, the theory developed here fails to apply; for much larger numbers, calculations become unnecessary.) Equation 50 is plotted in Fig. 11 for pulse numbers of 10 and 100 over the interesting range of detection probabilities. The plot demonstrates the rapid percentage increase of the false-alarm frequency for a 1% change in the detection probability when a very high detection probability is specified. Thus, a slight retreat from such stringent detection-probability specifications will yield a significant improvement in the false-alarm frequency keeping in mind that R^2 is about 10. On the other hand, a sacrifice of detection probability below 90% yields rapidly diminishing returns.

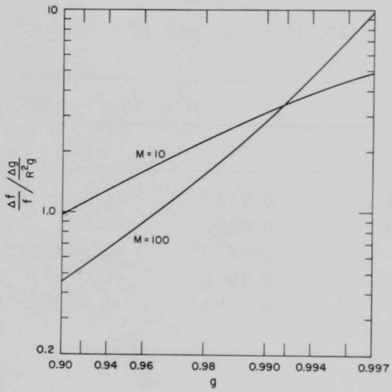

Fig. 11

Fractional Change in False-alarm Frequency per Unit Fractional Change in Detection Probability for Transients Consisting of 10 to 100 Pulses, as a Function of the Specified Detection Probability, at Unit S/N Ratio R

113-662

As regards the choice of time constant, T_b should be short enough to provide detection of the transient within a specified maximum delay. As discussed in Section II, the delay depends on the mission of the transient monitor; for situations of extreme hazard, the specifications might simply call for "as short a delay as possible." However, time constants of the order of magnitude of the transient length introduce considerable uncertainty in the detection probability, and even for somewhat longer time constants, Eq. 48 becomes unreliable.

Another matter of choice that can be guided to some extent by referring to Eq. 32 or 48 is the investment in detectors and ancillary equipment. Frequently, more detectors can be used, at only slightly worse detection efficiency, so that both M and N are approximately proportional to the number of detectors. One may then determine how many detectors will meet detection-probability and false-alarm specifications, provided the local background level is known. It may well turn out that the number of detectors exceeds the space available for them. In such a case, possible improvements are also available through the use of logic circuitry, which are discussed in Section VI.

Some remarks on the performance of a digital count-rate meter, as described in Section III, are presented in Appendix D.

VI. LOGIC PROCESSING OF TRANSIENT-DETECTION CHANNELS

The possibility of logic (coincidence) processing of transient-detection signals arises from the basic fact that many transient-detection systems use several detectors--partly for detection efficiency, partly for improving overall reliability. The detectors are frequently in a relatively inaccessible location, which may further be exposed to high temperature and radiation levels. The bulk of the channel electronics must thus be separated from the detectors. Even with careful shielding, cables interconnecting these units are prone to pick up noise pulses from nearby electrical machinery (pumps, motors, relays, etc.). Such pickup signals can be considerably reduced by requiring coincidence somewhere along the signal path.

A. Comparison of k-fold OR versus AND Scheme

Suppose now that the system disposes of k identical counters, operating at the same detection efficiency, and that a natural background at a level n exists in each channel. The question arises whether imposition of a k-fold coincidence requirement (or any less stringent coincidence requirement) increases or decreases the overall false-alarm frequency in comparison to the straightforward addition of all channels (as treated in Section V).

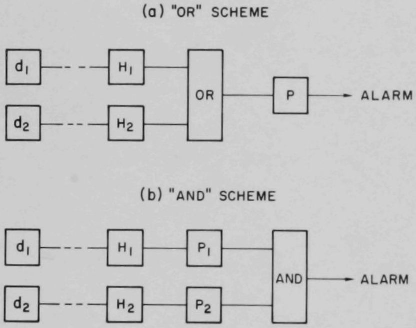

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Fig. 12. Logical OR versus Logical AND Arrangement of a Two-detector Transient Monitoring System. d_1, d_2 = detectors; H_1, H_2 = pulse amplifiers/pulse-height discriminators; P_1, P_2 = processors, consisting of count-rate meter, filter, and alarm discriminator operating in MNR mode; AND = coincidence circuit; OR = adder.

To clarify this discussion, Fig. 12 shows electronic block diagrams for both schemes of information processing. We shall, in the following discussion, denote the k-fold coincidence between alarm discriminator outputs as the AND system; we shall refer to the addition of all k channels ahead of the single CRM-alarm discriminator channel as the OR system. A transient of identical strength, consisting of M_1 pulses in the average, lasting a time T, is assumed in each individual channel.

To make a fair comparison between AND and OR logical connection of k channels, we shall assume that alarm levels are reset so as to secure equal detection probabilities according to Eq. 25, and then calculate the false-alarm frequency for either case. It is also interesting to compare false-alarm frequencies of each k-fold system with a single channel, with similarly specified equalization of alarm levels.

Let A_1 = alarm level (in single pulse heights) for a single channel, A_A = alarm level for each of k alarm discriminators in a k-fold AND scheme, and A_O = alarm level for the common alarm discriminator in a k-fold OR scheme. According to Eqs. 25' and 32, we can define the following ratios:

$$\left.\begin{aligned} W_1 &= \frac{A_1}{M_1} = 1 - G_1\sqrt{2/M_1}, \\ W_O &= \frac{A_O}{kM_1} = 1 - G_O\sqrt{2/kM_1}, \\ \text{and} \qquad & \\ W_A &= \frac{A_A}{M_1} = 1 - G_A\sqrt{2/M_1}. \end{aligned}\right\} \tag{51}$$

The parameters G_1, G_O and G_A are in turn related to the detection probability through Eq. 25 or 25". We shall prefer here the more convenient form given by Eq. 25":

$$g_i = 1 - B\exp(-pG_i); \qquad i = 1, O, A.$$

The condition of equal detection probabilities,

$$g_1 = g_O = g_A, \tag{52}$$

where, in view of the coincidence requirement for the AND scheme,

$$g_A = (g_1^*)^k = 1 - kB\exp(-pG_A) + \ldots, \tag{53}$$

yields the following relations between the parameters G_i:

$$G_1 = G_O = G_A - \epsilon, \tag{54}$$

where

$$\epsilon = (\log k)/p. \tag{55}$$

The detection probability g_1^* in Eq. 53 is that of each of the k channels in AND connection, whereas g_1 refers to one of these channels by itself, set to make $g_1 = g_A$.

The false-alarm frequencies can now be directly compared. For this comparison, which is expected to yield largely qualitative information, Eq. 32 (which lacks the Edgeworth correction) is entirely adequate. It is interesting here to investigate the effect of different choices of the time parameters T_i and T_b in relation to the expected transient duration T. Specifically, we consider three possible combinations:

$$\left.\begin{array}{l} (\alpha)\ T_b = T_i = T, \\[2ex] (\beta)\ T_b = T_i \gg T, \\[2ex] \text{and} \\[2ex] (\gamma)\ T_b \gg T_i. \end{array}\right\} \tag{56}$$

Sets α and β yield a single-channel false-alarm frequency:

$$\log f_1 = \log y - \log 2\pi T - \tfrac{1}{2} D^2 U_1^2 W_1^2; \qquad (T_i = T_b). \tag{57}$$

For γ, one finds, on the basis of Eq. 35, the similar expression

$$\log f_1' = \tfrac{1}{2} \log y + \tfrac{1}{2} \log z - \log 2\pi T - \tfrac{1}{2} D^2 U_0^2 W_1^2; \qquad (T_b \gg T_i); \tag{57'}$$

where the circuit response U_0 is given by Eq. 12. It is now apparent from Eq. 57' that a choice $T_i \ll T$ ($z \gg 1$) will result in considerably increasing the false-alarm frequency, as discussed near the end of Section IV. For almost all of the excursion monitors described in Section II, an announcement delay of the order of the mean transient duration T is acceptable. This leads to the reasonable specification $T_i = T$ in connection with the time-constant set γ, which will be adopted in the following:

$$(\gamma')\ T_b \gg T_i = T. \tag{56'}$$

The z term of Eq. 57' thus vanishes, while the last term of that equation is actually negligibly small ($T_b \gg T$; hence, $y \ll 1$ and $U_0 \ll 1$).

The time-constant sets are interrelated in two ways. On one hand, sets β and γ provide a comparison of two systems with equal values of T_b and T and different choices of T_i, which amount to maximum and minimum integration, respectively. On the other hand, sets α and γ', with fixed T_i and T, refer to minimum and maximum differentiation, bearing in mind that T_b is limited in length by practical considerations such as nonstatistical excursions and equipment stability, discussed in Section IV. The chosen time-constant sets thus span the full range of practically available two-element signal filters.

For k channels in OR connection, one readily finds

$$\log f_O = \log y - \log 2\pi T - \tfrac{1}{2} k D^2 U_1^2 W_O^2; \quad (T_i = T_b); \tag{58}$$

$$\log f'_O = \tfrac{1}{2} \log y - \log 2\pi T; \quad (T_b >> T_i = T). \tag{58'}$$

To compare the false-alarm frequency developed by an OR system and that developed by one channel of that system alone,† we combine Eqs. 57 and 57' with Eqs. 58 and 58' to get

$$\log f_O - \log f_1 = -\tfrac{1}{2} D^2 U_1^2 (k W_O^2 - W_1^2); \quad (T_i = T_b); \tag{59}$$

$$\log f'_O - \log f'_1 = 0; \quad (T_b >> T_i = T). \tag{59'}$$

These equations show, as discussed in Section IV, that the wrong choice of time constants can nullify any improvements of the performance arising from additional detector channels. The term in parentheses in Eq. 59 comes to

$$k W_O^2 - W_1^2 = (k - 1)[1 - 2(1 - W_1)/(1 + \sqrt{k})]. \tag{60}$$

For practical parameter choices, $(1 - W_1)$ ranges between 0.05 and 0.2; hence the correction term in Eq. 60 rapidly diminishes with additional count channels. The false-alarm frequency declines roughly exponentially with the number of OR channels, the slope being steepest for the choice of $T_i = T_b$ which maximizes the response U_1. This choice is practically equivalent to set α of Eq. 56.

Turning now to the AND system, we consider k identical channels with identically set alarm discriminators, each developing a gate pulse of length T_G whenever the alarm is tripped. Such events are rare enough to allow one to set T_G at will, without needing to consider channel paralysis effects. Alarms in each channel are thus distributed in a Poisson time series. The false-alarm frequency at the output of a k-fold coincidence circuit is then the equivalent of the "accidental" or "instrumental" coincidence rate, which for k channels comes to

$$\log f_A = k \log f_1^* + (k - 1) \log T_G + \log k$$
$$+ \log \left[1 - f_1^* T_G \left(\frac{k - 1}{k}\right) + (f_1^* T_G)^2 \frac{k - 1}{2(k + 1)} - \dots\right]. \tag{61}$$

†With alarm level readjusted to deliver the same detection probability for a standard transient of M_1 pulses that the OR system had for kM_1 pulses.

Equation 61 is readily derived from Poisson statistics. As for paralysis effects, the correction terms inside the brackets are negligible and may thus be dropped. Note that f_1^* is the false-alarm frequency in each channel when adjusted to yield a detection probability g_1^*.

To set the gates T_G, one evidently wants to choose the shortest possible length in order to minimize false alarms. A lower limit is imposed by time jitter, ranging from minimum through maximum announcement delay. The maximum announcement delay amounts to the risetime of a transient that is just barely detected, t_{max}, which is given by Eq. 34 for $T_i = T_b$; for $T_b \gg T_i = T$, $t_{max} \doteq T$. The minimum announcement delay, on the other hand, is small in comparison to the maximum, in view of the possibility of very large transients, which will trip the alarm early. Such considerations determine the practical gate length for the three sets of time constants as

$$\left.\begin{aligned} &(\alpha)\ T_b = T_i = T; && T_G \doteq T; \\ &(\beta)\ T_b = T_i \gg T; && T_G \doteq T_b; \\ &\text{and} \\ &(\gamma')\ T_b \gg T_i = T; && T_G \doteq T. \end{aligned}\right\} \tag{62}$$

Inserting Eqs. 57 and 57' into Eq. 61, and substituting the above gate-length choices, we obtain

$$\log f_A = B - \tfrac{1}{2} kD^2U_1^2W_A^2; \qquad T_b = T_i = T; \tag{63a}$$

$$\log f_A = B + \log y; \qquad T_b = T_i \gg T; \tag{63b}$$

$$\log f_A' = B + \frac{k}{2} \log y; \qquad T_b \gg T_i = T; \tag{63c}$$

where

$$B = \log k - \log(2\pi T) - (k-1)\log(2\pi). \tag{63d}$$

The set α of time constants, resulting in Eq. 63a, appears to result in a particularly strong reduction of false alarms through AND logic; set γ', Eq. 63c, is also interesting, whereas set β, Eq. 63b, is the equivalent of set γ for $k = 2$, but falls behind for $k > 2$.

As before, we may compare the AND scheme to an equivalent (equally efficient) single channel:

$$\log f_A - \log f_1 = \log k - (k-1)\log 2\pi - \tfrac{1}{2}D^2U_1^2(kW_A^2 - W_1^2); \quad T_i = T_b = T; \tag{64}$$

$$\log f'_A - \log f'_1 = \log k - (k-1)\log 2\pi; \quad T_b \gg T_i = T \text{ and } T_b = T_i \gg T. \tag{64'}$$

The second quantity in parentheses in Eq. 64 comes to

$$kW_A^2 - W_1^2 = (k-1)\, W_1^2\left[1 - \frac{2k}{k-1}\sqrt{\frac{2}{M}}\,\frac{\epsilon}{W_1} + \frac{k}{k-1}\,\frac{2}{M}\left(\frac{\epsilon}{W_1}\right)^2\right]. \tag{65}$$

The second and particularly the third terms inside the brackets of Eq. 65 are small (at least for a number of channels less than about 10). The choice of short time constants is therefore an effective means of reducing false alarms in conjunction with AND logic, just as for OR logic. This reduction is, however, less effective than in conjunction with OR logic, as may be seen by comparing Eqs. 60 and 65. In contrast, AND logic with the other sets of time constants still reduces false alarms, whereas OR logic does not (cf. Eqs. 59' and 64'). These facts may be exploited as discussed in Section B below.

The direct confrontation of OR and AND logic, again with alarm settings that equalize the detection probability, yields

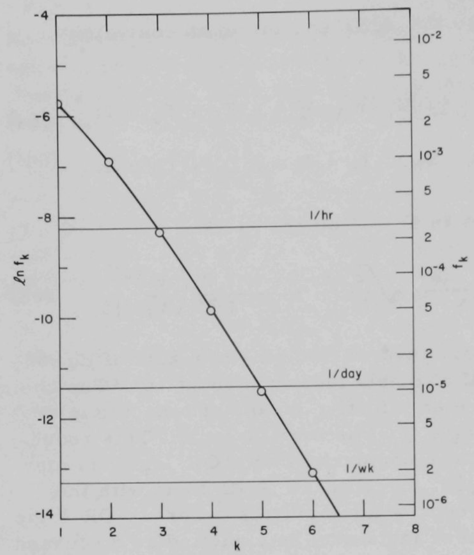
$$\log f_A - \log f_O = \log k - (k-1)\log 2\pi - \tfrac{1}{2}kD^2U_1^2(W_A^2 - W_O^2). \tag{66}$$

where the second term in parentheses is clearly negative. According to Eq. 51,

$$k(W_A^2 - W_O^2) = -2kW_1\sqrt{2/M_1}\left[G_1(1 - 1/\sqrt{k}) + \epsilon + [G_1^2(1 - 1/\sqrt{k})^2 - \epsilon^2]\big/ W_1\sqrt{2M_1}\right]. \tag{67}$$

Equation 66 indicates that the OR connection becomes increasingly preferable to the AND connection as the number of channels is increased; this conclusion is expected to be reasonably valid over a certain range of time-constant choices not too different from set α. In contrast, AND logic can apparently produce beneficial results for long time constants T_b, in which case OR connection would seem to be a mistake. To emphasize the latter point, Fig. 13 shows a plot of the false-alarm frequency for AND logic against the number of channels, k, given time-constant set β and T_b = 50 sec. Acceptable values are evidently reached with k equal to 5 to 6.

To sum up the results of this section in practical terms, we shall suppose that a large number of equivalent count channels are somehow



113-660

Fig. 13. False-alarm Frequency for k-fold AND Logic and a Choice of Time Constant T_b = 100 T (T = transient duration). To guide the eye, a line has been drawn through the points corresponding to integers k. This line has a slope approaching $-\log(2\pi)$ asymptotically.

available, and further specify a rather low channel count rate. For one or two such channels, it may then, in fact, not be possible to make any valid prediction or estimate of the count rate when the time constants, particularly T_b, are made comparable to the duration of an expected short transient (time-constant set α). As more channels are added in parallel (i.e., OR logic), an equation similar to Eq. 58, but with an Edgeworth correction term, can be reasonably applied. Eventually, Eq. 58 should become valid. A similar exercise with long clipping times T_b would be within the range of statistics (if possibly with an Edgeworth correction) at a correspondingly lower total rate, i.e., when only a few counters are used. When channels are then added at the input of the count-rate meter, statistical theory, which is unquestionably valid here, predicts that there is no effective improvement of the false-alarm frequency. The same total input rate, deployed in AND configuration, results in reducing the false-alarm frequency exponentially with the number of channels.

If cost is not important in some situation where performance of the highest quality is wanted, and sufficient stability can be designed into the electronics, an AND system is thus advantageous in connection with large values of T_b, where T_i may be set to some value between T and T_b as a practically acceptable compromise between announcement delay and false-alarm frequency. Such a policy would be indicated particularly for situations in which the duration of transients is uncertain within wide limits. When transient duration is determined by flow or processing speed and therefore relatively well known, a choice of time constants close to that duration, with maximum deployment of OR-added detection channels, will give the lowest false-alarm frequency at some acceptable detection probability. These results are perhaps worth pointing out; since the effect of signal filtering on the performance of single channels is often overlooked, such effects tend to be neglected a fortiori in connection with logic processing.

Before leaving this subject, we will add some remarks concerning the possibilities inherent in combinations of AND systems with different sets of time constants.

B. Composite Logic Circuits

In nuclear coincidence spectrometry, accidental coincidences can be considerably reduced through so-called "fast-slow" or "slow-fast" logic. These logic systems basically consist of four channels connected to two detectors viewing the same source; each detector feeds into one "slow" and one "fast" channel. Coincident events in the source result in pulses in the respective fast channels, which are precisely timed but have a poor correlation between pulse height and event energy release; in contrast, the slow channels deliver poorly timed pulses, whose height contains the desired energy information. This is effected by integrating filters in the slow channels, through which the S^2/N ratio is increased as described in Section III. As a result, pulse-height discriminators in the slow channel exhibit considerable time jitter, and slow coincidence gates must be made at least as long as this jitter (as discussed in Section A above). If, however, fourfold coincidences between slow- and fast-channel outputs are required, accidental coincidences can occur only when two independent source events deliver pulses of appropriate height within the fast gates. The fourfold coincidence requirement is usually imposed in two stages, combining fast and slow twofold in each channel and recombining channel outputs in a final adder, or else adding fast and slow channels separately and then adding outputs of these stages. The designations "slow-fast" and "fast-slow" have become customary for these arrangements.

The analogy with pulse processing, which was mentioned in Section III, also carries over for coincidence logic. Thus, "slow" channels have time constants $T_b = T_i >> T$ (set β); "fast" channels may have time-constant sets α or γ. One possible arrangement of the logic system is shown in Fig. 14. Fast channels here are simply derived from the "raw"

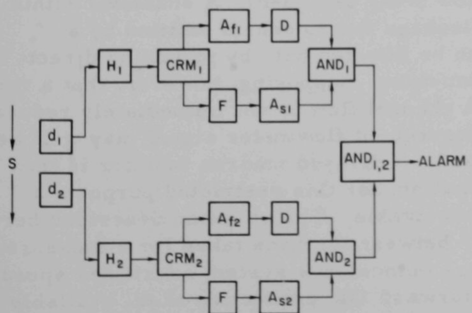


Fig. 14

Block Diagram of "Slow-Fast" Version of Twofold, Two-stage AND Logic. d_1, d_2 = detectors, of closely similar characteristics; H_1, H_2 = detector-pulse processors (amplifiers/discriminators); CRM_1, CRM_2 = count-rate meters, with time constants T_b; A_{f1}, A_{f2} = "fast" alarm discriminators operating in MNR mode and scanning the raw CRM trace; A_{s1}, A_{s2} = "slow" alarm discriminators, also operating in MNR mode; F = filters of time constant T_b; D = delay of approximately T_b; AND_1, AND_2 = coincidence adders with output pulse width T; and $AND_{1,2}$ = final coincidence adder.

output trace of count-rate meters, before passing this trace through an integrating filter. (In practice, a filter $T_i = T$ may still be inserted.) The occurrence of a fluctuation that trips the slow channel will then always also trip the corresponding fast channel, since integrating filters can only smooth, but not add excursions. The smoothing effect is apparent from a comparison of Eqs. 57 and 57'. In the system shown in Fig. 14, which is of "slow-fast" type, each detector channel delivers a rate given by Eq. 57' to the final coincidence circuit, but coincidence gates are kept to length T, delayed by T_b in order to encompass jitter in the slow channels. The resultant false-alarm frequency delivered by the final coincidence stage comes to

$$\log f''_A = B + k \log y. \qquad (63e)$$

This frequency is evidently very much lower than either Eq. 63b or 63c would predict for one-stage coincidence systems. The central advantage of a two-stage scheme lies in its use of only a modest number of detector channels. For example, let $y = 0.01$, where $T = 0.5$ sec and $T_b = 50$ sec. Then two-stage AND logic processing yields a false-alarm frequency of about 3×10^{-6}, or once every four days, corresponding to five or six slow channels from independent detectors (of Fig. 13). There is evidently a net saving of equipment, aside from the possible difficulty in accommodating six detectors in a scanning head or probe for certain types of transient monitors discussed in Section II.

The system described here still has two inherent drawbacks: First, there is a considerable premium on alarm-level stability, and second, the announcement is necessarily delayed by the integrating time in the slow channels (in the quoted example, by 50 sec). This makes such a system unsatisfactory for an application in which the occurrence of a transient requires immediate remedial action, such as fuel-failure warning in certain reactor plants. Such reactor plants may, however, not even allow the unavoidable delay due to the transport of fission products from the reactor core to the detection station (which requires a minimum time of about 5 sec in a number of such plants now being designed). A shutdown within about 1 sec after a core channel blockage due to debris emitted by a "catastrophic" fuel failure can then be effected only by providing direct-reading instrumentation, e.g., flowmeters. Supposing, however, that a relatively mild failure occurs in which channel flow is not immediately reduced (or is only slightly reduced); the consequent flowmeter signal may well be so small that it can be easily missed. A delayed neutron monitor is then still very useful to provide confirmation; for this restricted purpose, a delay of 50 sec should be readily acceptable. The situation described here is an example of a general relation between the time taken for a measurement and its precision. A two-stage coincidence system sacrifices speed for reliability, whereas a straightforward OR connection of all available channels with time-constant set α tends to be unreliable (unless the total count rate is high enough), but provides speed.

Another example in which two-stage AND logic is readily applicable is an area survey, as discussed in Section II. Here, it may at first appear that the delay inherent in such a system will result in associating the wrong coordinates with the event. Note, however, that this delay is precise and thus results in an offset of equivalent precision which can be allowed for in the registration device.

To sum up, composite logic processing can reduce false alarms considerably and thus merits serious consideration where a precise announcement delay can be tolerated.

VII. SUMMARY

A. Theory

This report discusses the use of relatively simple electronics equipment to detect count-rate transients in a channel that normally delivers random "background" counts at a quasi-constant mean rate. The basic problem in this task is the reliable distinction between such transients and statistical excursions of the background, a matter essentially of exercising judgment in the visual inspection of a record delivered by the equipment.

In favorable cases, judgment is improved through experience and becomes more reliable as the "signature" (the shape of the transient characteristic of the causative chain of events that results in the count-rate excursion) becomes established. For these situations, the equipment may only be required to produce a record in which transient shape is readily recognizable and amplitude is enhanced relative to background excursions, as considered in Section III. More generally, another equipment function is required: The transient must be announced, possibly in addition to being displayed. The latter requirement has certain quantitative implications, discussed in Sections IV and V. In marginal situations, the performance of the alarm discriminator may be improved by logic processing of the outputs of several channels, as discussed in Section VI.

The readability of the display is in part a matter of setting the time constants of the processor so as to maximize the S^2/N ratio of the channel; in part, more qualitative considerations come into play. In a straightforward treatment of the S^2/N ratio, the signal may be taken as the peak response to a standard square transient of duration T, while the noise is the rms background fluctuation calculated by Campbell's theorem. The resultant equation suggests that at least one of the filter time constants be made comparable to T. Moreover, the readability of complex transients also improves as the CRM or input time constant (which plays here the role of the input RC circuit of a nuclear detector and preamplifier) is made comparable to the transient duration. This may not be fully realized by operating personnel, particularly since count-rate meter channels required to detect <u>slow</u> variations in the input rate demand the <u>longest</u> possible time constant. Since count-rate meters are generally used to detect slow rate variations, they are often equipped with a switch labeled "percent error." For transient detection, this designation must be ignored. Moreover, the set of time constants provided in the instrument may have to be changed, and it is further desirable to provide a second or integrating time constant (which improves the S^2/N ratio somewhat and is important in connection with the alarm discriminator discussed in Sections IV and V). Finally, a second clipping stage is needed to make alarm-discriminator performance independent of background. In certain transient-detection situations, it may be expedient to provide two independent processor channels, fed by the same detectors, with somewhat different filtering. Each channel can thus be optimized for a transient of certain mean duration.

The equations giving the S^2/N ratio are valid only as long as Campbell's theorem (through which the mean background fluctuation is calculated) applies. As discussed in Sections III-VI, this theorem is meaningful only for large values of the statistic nT_b, where T_b = equipment time constant and n = mean background rate. Evidently, situations can occur in transient monitoring where this statistic is small when T_b has been set to be comparable with T. This does not mean that one should then use a longer time constant, or that one should not make every reasonable effort to reduce the background. It does suggest the possibility of increasing the number of detectors or the intensity of the source (for a stimulated response system) in order to achieve a more <u>reliable</u> quality of performance.

When an alarm discriminator is used to alert personnel to the occurrence of a transient or to record transients automatically (as in an area sweep-survey system), the choice of alarm level is added to the choice of filter time constants. Moreover, the semiqualitative criteria of S^2/N ratio and signature recognizability are replaced by more quantitative, if statistical, performance parameters: false-alarm probability per unit time, detection probability, and mean delay between onset of the transient and alarm. Optimization of the equipment variables implies a tradeoff between these interrelated detection-quality parameters. The confidence with which one may calculate the detection probability is largely predicated on the degree of correspondence between the model on which the calculation must be based and real transients. As regards the false-alarm frequency, a first survey, presented in Section IV of this report, yields a straightforward formula, strictly valid only for background rates that may be somewhat unrealistic. This is due to the dependence of this formula on the distribution of the trace <u>slope</u> as well as of the trace level, which makes the false-alarm frequency even more sensitive to the statistic nT_b than the S^2/N ratio, considered above. Within this rather limited range of applicability to practical transient-detection problems, the false-alarm frequency, when plotted against the filter time constant at a fixed detection probability, shows a spectacular dip for small time constants, where the improvement in S^2/N ratio makes itself felt. This choice of time constants also corresponds to minimum delay time and thus might be considered mandatory where the transient is connected with a serious hazard. The false-alarm frequency also decreases more gradually for very long time constants, due to the decreasing <u>frequency</u> of background excursions, which for those time constants may have amplitudes considerably in excess of the mean excursion due to a genuine transient.

To extend the range of validity of the false-alarm frequency formula toward practically encountered background rates, more realistic trace level and slope distributions must be used in false-alarm frequency computation (which uses, in the first approximation, Gaussian distributions for both level and slope). Distributions that do not differ strongly from Gaussian shape can be described in terms of the product of a Gaussian and a corrective series, such as the Gram-Charlier series or the Edgeworth series, of which the latter has the advantage of allowing a straightforward evaluation in terms of the filter response. On the strength of this analysis, one may predict the false-alarm frequency with more confidence within a readily defined statistical limit. Moreover, the dependence of the relatively strong first correction $A(d^3\phi/d\beta^3)$ term on the symmetry of the pulse shape delivered by the filter suggests the possibility of canceling this term by special filtering and thus achieving, at least over a limited range of time constants, a lower false-alarm frequency at given detection probability.

This possibility of overall quality improvement is roughly analogous to certain types of smoothing functions, which can be shown to improve the retrieval of a recurrent signal buried in noise, a subject that has received considerable attention in recent years.[22] In the present context, it amounts to reducing or removing all odd moments of the distribution. The first moment is removed by any filter that blocks dc components; the above-mentioned dominant contribution to the false-alarm frequency in the region of interest is due to the third moment.

Certain analogies with the corresponding problem of background elimination in nuclear counting work further suggest that the false-alarm frequency can be reduced through logic processing of independent outputs of several channels. A closer look at this possibility, presented in Section VI, discloses that a straightforward coincidence requirement* results in a higher overall false-alarm frequency than the addition of all channels ahead of a single alarm discriminator, provided that minimum time constants are chosen. Such a logic scheme does not, however, use all the information available from the equipment, considering both amplitude and time dependence of trace excursions. An arrangement similar to a "fast-slow" coincidence system can take advantage of the relatively low false-alarm frequency of an alarm discriminator sitting on a filtered trace, and the sharp timing of alarms derived from unfiltered traces, to result in a worthwhile overall reduction of false-alarm frequency.

The possible improvements of alarm-discriminator performance through response pulse shaping or through the more elaborate means of logic processing are, at this stage, only suggestions that remain to be tested. In fact, the basic relations between count rate, time constants,

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*Between some arbitrary number of alarm discriminators fed by independent and equivalent detector channels (all viewing the same source of background and transients).

alarm level, and false-alarm frequency given in this report are equally in need of experimental verification, for which purpose an experiment has been started. A count rate one to two orders of magnitude higher than the practically encountered range is used in this test, with appropriate scaling of time constants. Even so, false alarms in the upper range of level settings are accumulating at a slow rate, and the experiment must consequently continue over a long period before statistically significant information is available. Any schemes for further improvement must await these results. In a practical sense, these schemes can be considered seriously only when all other means, such as increasing the number of detectors and improving the geometry, have been implemented but still fail to provide adequate performance. The cost of such measures is evidently justifiable if it can be shown that failure to detect a genuine transient may result in a very serious health hazard, equipment failure, or other untoward event, while, on the other hand, a high false-alarm frequency results in an equally intolerable burden on operating personnel and waste of time. Fuel-failure detection in fast reactors is at least one application of transient detection where a fairly high investment and development cost is not unreasonable.

B. Applications

In Section II, several different transient monitors were mentioned and partially described; we add here only some remarks regarding the applicability of the theoretical framework developed above to some particular systems. Sweep-type systems, which search for a steady source (or stimulated emission) by displacement of the detector, allow some latitude in parameter adjustment and are generally able to accommodate a relatively high false-alarm frequency, especially if the search can be readily interrupted for a closer second look. These are, however, exceptions such as track-plate scanning and related scanning tasks where stopping is not allowed, since the equipment normally runs unattended. The purpose of such an instrument is basically connected with a desire to save time and effort. The automatic scanner competes against the skill of personnel trained in track scanning, and thus must offer speed and reliability at a reasonable equipment cost. The processor time constant for scanning must be comparable to, or shorter than, the duration of the transient, as determined by sweep speed and aperture, so as to provide a direct means of determining the location on a clock device that is coupled to the sweep (for example, a scaler counting interferometer fringes, an interferometer being mounted on the plate carriage). The "background" is adjustable here through the intensity of illumination; the transient is a reduction rather than an increase in the count rate.

The duration of transients due to tracks is generally not fixed; it may be one of the pieces of information sought. The equations presented in this report thus require some modification to extract relations from which to find optimum settings for adjustable parameters in this problem: the sweep speed, illumination, processor time constants, and discriminator level. For those settings, a false indication probability for the whole sweep, and a corresponding nondetection probability, can be estimated. This example of a transient monitor is evidently similar to area scans in medicine and astronomy.

A final remark concerns the fuel-failure monitor, frequently mentioned throughout this report and described in Section II. Such systems are often only considered after reactor plant construction is well on its way, and thus may have to make do with less than favorable conditions. Evidently the most practical step toward improving the overall performance of this type of transient detector would be an early consideration of its requirements, first and foremost the interpretation of the transient strength and shape in terms of type, extent, and perhaps approximate location of a fuel failure. In existing installations, such cause-and-effect relationships are largely indeterminate.

This uncertainty is due in part to the vagaries of the flow pattern, discussed elsewhere,[17] and in part to lack of information concerning the details of the cladding rupture itself. The assumption made in this report, that fission products are highly localized as they pass through the detector, implies a certain flow pattern, as well as very rapid release of the contamination. For an unfavorable flow pattern,* timely detection of fuel-cladding rupture may be difficult. Finally, a coolant channel may be partially or entirely blocked shortly before rupture occurs, in which case the injection of fission products into the main coolant stream would yield a considerably attenuated transient signal.

These remarks point to the need for flow-pattern studies and similar work in support of a realistic program of fuel-failure indication improvement, together with concurrent development in signal processing.

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*The flow pattern may vary considerably over the core face, such that a slug of contamination originating in certain regions is dispersed in turbulent eddies while a similar cladding burst elsewhere results in a strong signal.

VIII. SYMBOLS, FORMULAS, AND CONCLUSIONS

The formulas derived in the text and in the appendixes are collected here for easier reference, and principal inferences and conclusions are briefly restated. Subsections are labeled with the names of the sections to which they refer.

A. Introduction

A large variety of different types of instruments can be classified as transient detectors, on the basis of the following common features: (1) Input is a random count, Poisson distributed in time, with superposed transient rate increases of short duration; (2) these transients are generally rare and occur unpredictably; and (3) the purpose of the instrument is to detect such transients with high reliability.

Some of these instrument systems are required to record and display transients, possibly with enough detail to allow recognition of the "signature" of the event that caused the transient. Other instruments have the primary purpose of warning, usually within a short announcement delay. For the general case, certain specific methods of processing the input will (a) provide the best display; and/or (b) provide the most reliable warning. These methods are discussed here.

B. Specific Transient-detection Systems

Transient-detection equipment may be conveniently classified according to function. One kind of transient detector searches an area; a transient occurs whenever the scan passes over a source. However, this transient detector type is not emphasized in this report, which is mainly concerned with the other kind of transient detector, a stationary surveillance instrument; the occurrence of a transient signifies a malfunction or other untoward event. The prime example of this kind of transient monitor, considered in some detail here, is a fuel-failure monitor used in connection with a high-power-density reactor plant.

C. Response of the Processor

We suppose that pulses randomly spaced in time are applied to the input of a channel consisting of a count-rate meter with certain additional filters, each filter being described by a time constant. Normally, the CRM time constant is the equivalent of a differentiator with time constant T_b. A following smoothing filter or integrator has time constant T_i; and a final clipper, which may or may not be present, has a time constant labeled here T_a. More complex filters are not discussed. Any differentiating time constant that is very much longer, or integrating time constant very much shorter, than other circuit time constants can be neglected; the circuit will respond as if this filter element were absent.

The combined filter delivers a continuous voltage or current analog of the input rate, modified by the filter response function. We now suppose that there are occasional transients in the input rate, of well-defined duration and intensity. The object of the filter is to portray such transients in a form that allows them to be readily distinguished from the natural statistical fluctuations in the normally prevailing background rate. Let

n = input mean background rate,

M = number of input pulses in the transient, quasi-uniformly spaced,

T = duration of the transient,

T_b = CRM or first-differentiation time constant,

T_i = integration time constant,

T_a = second-differentiation time constant,

$y = T/T_b$,

$z = T/T_i$,

$D^2 = M^2/N$,

and

$N = nT$.

The quality of the presentation is determined by the ratio of peak response to the transient to the square root of the second moment, or variance, of the trace due to background alone. The latter is supposed to be calculable through Campbell's theorem. Input pulse length is assumed negligible in comparison with any other time parameters. Let

H_{max} = peak transient response,

σ^2 = second moment of the trace,

and

$$R^2 = H_{max}^2/\sigma^2.$$

Then, for arbitrary T_b and T_i, and $T_a \gg T_i$ and T_b,

$$R^2 = 2D^2(y^{-1} + z^{-1})(e^y - 1)^{2z/(z-y)}(e^z - 1)^{2y/(y-z)}. \tag{17}$$

As a further special case, let $T_i \ll T_b$; then,

$$R^2 = \frac{2D^2}{y}(1 - e^{-y})^2 = 2D^2/h^2y = D^2U_0^2. \tag{10}$$

Another important case, $T_i = T_b$, yields

$$R^2 = 4(D^2/h^2y) \exp[-2y(h-1)] = D^2U_1^2, \tag{17'}$$

where the function

$$h = (1 - e^{-y})^{-1}, \tag{11}$$

and the response functions

$$U_0 = \sqrt{2}/h\sqrt{y} \tag{12}$$

and

$$U_1 = (2/h\sqrt{y}) \exp[-y(h-1)] \tag{20}$$

have been introduced. Further defining

$$Z^2 = 1 + y^2h(h-1), \tag{21}$$

we find a response for three equal time constants, $T_a = T_b = T_i$,

$$R^2 = D^2U_2 = (16D^2/h^2y)\{\exp[-2y(h-1)]\}\{(Z-1)^2 \exp[2(Z-1)]\}, \tag{22}$$

hence

$$U_2 = 2U_1(Z-1) \exp(Z-1), \tag{23}$$

which is derived in Appendix B. Equations 12, 20, and 23 are plotted in Fig. B.1.

Finally, consider the response for a digital channel in which square pulses of length T_b are produced for each input; this comes to

$$\left.\begin{aligned} R_d^2 &= D^2y, \quad && y \le 1, \\ \text{and} \quad R_d^2 &= D^2/y, \quad && y \ge 1. \end{aligned}\right\} \tag{24}$$

The following conclusions are pertinent:

1. Each of these response functions peaks for values of T_b near T, i.e., for values of y near unity. The actual peak values do not differ much, amounting to 0.8 to 0.9 times the detectivity D; a single differentiation and equal integration are about best.

2. When the expected duration T is shorter than the electromechanical integration time constant of the chart recorder used in display, the resultant record is necessarily partly filtered, even when the equipment is otherwise just a stock count-rate meter with single time constant T_b. Adequate display further implies another problem: The paper feed rate must be high enough to spread the transient peak over, say, several inches. These problems evidently require special solutions.

3. The customary use of count-rate meters for displaying slowly varying count rates often leads to the use of long count-rate meter time constants T_b for transient monitors. The equations giving R^2 as a function of time parameters indicate that this is a poor choice.

D. Performance of the Discriminator at High Background Rates

If we now further suppose that the channel output is made to trigger an alarm whenever the trace exceeds some preset value, the quality of the presentation can be described in an entirely quantitative way through the detection probability, false-alarm frequency, and maximum announcement delay. To overcome the effect of slow variations in the background, a final clipper is used in the output stage of the filter, making the alarm trigger a mean-noise-reference (MNR) instrument. Let

g = detection probability,

A = number of pulses that have to pile up to trigger the alarm,

and

f = false-alarm frequency.

Then, in the limit, where the distribution of transient counts can be described by a Gaussian with mean and variance $M + N$, one can define the detection probability for an MNR alarm discriminator by

$$g = \tfrac{1}{2}[1 + \Phi(G)] \sim 1 - Be^{-pG}, \tag{25''}$$

where

$$G = (M - A)/(2M)^{1/2}. \tag{26'}$$

We suppose that M can be estimated from calibration tests and is thus a known parameter. The constants

$B = 1.90$

and

$p = 3.17$

give a reasonable fit over the useful range $0.90 \leq g \leq 0.99$.

Under conditions where A has been selected to obtain a certain value of g, in the limit of large values of nT_b throughout the range of T_b, and with equal time-constant filtering,

$$\log f = \log(y/2\pi T) - \tfrac{1}{2} D^2 U_1^2 W^2, \tag{32}$$

where the parameters T, y, D, and U_1 have been defined above and

$$W = 1 - (2/M)^{1/2} G. \tag{33}$$

The relationship between these various parameters can be judged from the following table of limiting values:

A	G	g	W	f
0	$(M/2)^{1/2}$	1	0	$y/2\pi T$
M	0	0.5	1	$(y/2\pi T)\exp(-D^2U_1^2/2)$

The case of different filter time constants ($T_i < T_b$) is briefly described in Section VI, which leads to the conclusion that the performance is worse with short integration (but can be improved, as considered in Section V, with pulse symmetrization).

The following general conclusions apply:

1. The false-alarm frequency for a chosen detection probability can be calculated with confidence from Eq. 32 only when $nT_b \gg 1$. That equation then predicts a spectacular reduction in f for $T_b \approx T$. Such a choice of T_b therefore implies that $nT \gg 1$ to make Eq. 32 applicable; when this is not the case, the false-alarm frequency cannot be estimated reliably.

2. This breakdown of statistical theory does not necessarily imply that large time constants should be preferred. In the large T_b range, Eq. 32 is valid and describes a situation that is inadmissible in most practical applications of transient monitoring. Let T_f = mean time between false alarms; then $T_f = 2\pi T_b$ in the limit of very long time constants. We must now consider that T_b is also the maximum announcement delay. Moreover, the response to real transients shrinks, for such large values of T_b, to a level comparable to the unavoidable alarm-level jitter due to instrument noise.

3. Intermediate or compromise values of T_b can easily result in particularly high values of f. Unfortunately, these are frequently the values available in stock count-rate meters.

E. False-alarm Frequency for Practically Encountered Background Rates

The condition that nT_b be large over the entire range of time constants, down to $T_b = T$, is often not met in practical transient-detection problems. Equation 32 becomes increasingly inexact with declining values of T_b (corresponding to increasing values of y). A somewhat more reliable formula can be found on the basis of Edgeworth's series. One finds that

$$\log\frac{f}{n} = \log(u/2\pi) + \log[1 - E(u)] - \tfrac{1}{2}H^2u[1 - 2\epsilon^2u^2 + 2.11\epsilon^4u^4 - \ldots], \tag{48}$$

where

$$H = (2M/e)[1 - (2/M)^{1/2}G], \tag{46}$$

and

$$\epsilon^2 = N^2/24. \tag{47}$$

Further,

$$E(u) = \sum_{i=1}^{\infty} k_i u^i \tag{49a}$$

is the "Edgeworth correction" in terms of the parameter

$$u = (nT_b)^{-1} = y/N. \tag{44}$$

The coefficients $k_1, \ldots, k_5$ are functions of H, N, and the type of filter used; for $T_b = T_i$, they are

$$\left.\begin{aligned}
k_1 &= 3(Ha+b) + c,\\
k_2 &= H^3a + H^2d + 3(Ha+b)\,c,\\
k_3 &= H^4e + 3Ha\epsilon^2 + (H^3a + H^2d)\,c,\\
k_4 &= H^6a^2/2 - 6H^2d\epsilon^2 - 3H^3a\epsilon^2 + (H^4e + 3Ha\epsilon^2)\,c,\\
&\text{and}\\
k_5 &= \epsilon^2[4H^4e - 3.3Ha\epsilon^2] + H^6a^2/2 - 6H^2d\epsilon^2 - 3H^3a\epsilon^2)\,c,
\end{aligned}\right\} \tag{49b}$$

with numerical constants

$$\left.\begin{aligned} a &= 0.0987654, \\ b &= 0.0087615, \\ c &= 0.0195973, \\ d &= 0.1257285, \\ e &= 0.0575345, \\ &\vdots \end{aligned}\right\} \quad (49c)$$

derived from Campbell's theorem.

To the general conclusions of Section D above, we add here:

1. Equation 48 is useful only within a factor of about two, i.e., for corrections that add up to unity. Beyond that, the statistical basis of some of the assumptions that allowed the derivation of this equation begin to fail.

2. Equation 48 is derived from a modified Gaussian distribution, containing both even (symmetric) and odd (antisymmetric) correction terms. The latter terms, which would change sign for an alarm level below mean noise, must vanish when the input pulse response of the filter is symmetric. However the symmetry must be fairly precise, so that the component of the pulse appearing below zero is an exact delayed replica of the pulse component above zero. A filter or set of filters that yields such a response would therefore improve the performance of a transient detector and is recommended. (Details are presented in Appendix B.)

F. Logic Processing of Transient-detection Channels

In considering the effect of certain combinations of independent channels, we suppose that k identical detectors are available, and let

M_1 = number of signal inputs in each channel;

g_1 = detection probability in each channel, when equipped with alarm set at a parameter G_1 (defined below);

and

f_1 = false-alarm frequency in each channel.

The following cases are now described:

1. All channel inputs are combined and fed to a single processor. This processing is called the OR scheme. Let

kM_1 = signal pulse number (OR);

g_O = detection probability (OR), at a parameter G_O (defined below);

and

f_O = false-alarm frequency (OR).

2. Channels are processed by identical processors, and gates generated by "alarms" in the processor outputs are applied to a k-fold coincidence circuit, whose output in turn triggers the overall (actual) alarm. For this processing method, called here the AND scheme, let

g_A = detection probability (AND),

and

f_A = false-alarm frequency (AND).

To make a fair comparison between a single channel, a k-fold OR system, and a k-fold AND system, we impose the condition that detection probabilities be set equal for these three situations,

$$g_1 = g_O = g_A, \tag{52}$$

and compare resulting false-alarm frequencies. This comparison further requires specification of time constants, for which we select three representative sets

$$\left.\begin{aligned} &(\alpha)\ T_b = T_i = T, \\ &(\beta)\ T_b = T_i \gg T, \\ \text{and} \\ &(\gamma)\ T_b \gg T_i = T. \end{aligned}\right\} \tag{56}$$

The detection probabilities g_i are expressed in terms of parameters G_i by

$$g_i = 1 - B\exp(-pG_i); \quad i = 1, O, A; \tag{25"}$$

which further leads to the definition of useful quantities W_i:

$$\left.\begin{aligned} W_1 &= A_1/M_1 = 1 - G_1\sqrt{2/M_1}, \\ W_O &= A_O/kM_1 = 1 - G_O\sqrt{2/kM_1}, \\ \text{and} \\ W_A &= A_A/M_1 = 1 - G_A\sqrt{2/M_1}. \end{aligned}\right\} \tag{51}$$

The numbers A_i are settings of respective alarm discriminators, corresponding to the number of pulses contained in a transient that is just barely accepted (any number $\geq A$ triggers the alarm). From Eqs. 51 and 25", we further obtain the useful relation

$$G_1 = G_O = G_A - (\log k)/p. \tag{54}$$

We then find the single-channel false-alarm frequencies

$$\left.\begin{aligned} \log f_1 &= -\log 2\pi T - \tfrac{1}{2} D^2 U_1^2 W_1^2, && (\alpha) \\ \log f_1 &= \log y - \log 2\pi T, && (\beta) \\ \text{and} \\ \log f_1 &= \tfrac{1}{2} \log y - \log 2\pi T. && (\gamma) \end{aligned}\right\} \tag{57}$$

For the OR system, similarly,

$$\left.\begin{aligned} \log f_O &= -\log 2\pi T - \tfrac{1}{2} k D^2 U_1^2 W_O^2, && (\alpha) \\ \log f_O &= \log y - \log 2\pi T, && (\beta) \\ \text{and} \\ \log f_O &= \tfrac{1}{2} \log y - \log 2\pi T. && (\gamma) \end{aligned}\right\} \tag{58}$$

For the AND logic system,

$$\log f_A = k \log f_1^* + (k-1) \log T_G + \log k, \tag{61}$$

where now

$$\log f_1^* = -\log 2\pi T - \tfrac{1}{2} D^2 U_1^2 W_A^2, \quad (\alpha);$$

and

$$\log f_1^* = \log f_1, \quad (\beta) \text{ and } (\gamma).$$

The coincidence gate T_G is chosen to cover the timing uncertainty:

$$\left.\begin{aligned} T_G &= T, && (\alpha) \text{ and } (\gamma); \\ T_G &= T_b, && (\beta); \end{aligned}\right\} \tag{62}$$

resulting in AND channel false alarms at the rates

$$\left.\begin{aligned} \log f_A &= B - \frac{k}{2} D^2 U_1^2 W_A^2, && (\alpha) \\ \log f_A &= B + \log y, && (\beta) \\ &\text{and} \\ \log f_A &= B + \tfrac{1}{2} \log y, && (\gamma) \end{aligned}\right\} \qquad (63)$$

where

$$B = \log k - \log 2\pi T - (k-1) \log (2\pi).$$

We may now compare first the OR and single-channel cases:

$$\left.\begin{aligned} \log f_O &= \log f_1 - \tfrac{1}{2} D^2 U_1^2 (k W_O^2 - W_1^2), && (\alpha); \\ &\text{and} \\ \log f_O &= \log f_1, && (\beta) \text{ and } (\gamma); \end{aligned}\right\} \qquad (59)$$

where

$$k(W_O^2 - W_1^2) = (k-1)[1 - 2(1 - W_1)/(1 + \sqrt{k})]. \qquad (60)$$

Similarly, comparing AND and single-channel cases yields

$$\left.\begin{aligned} \log f_A &= \log f_1 + \log k - (k-1) \log (2\pi) - \tfrac{1}{2} D^2 U_1^2 (k W_A^2 - W_1^2), && (\alpha); \\ \log f_A &= \log f_1 + \log k - (k-1) \log (2\pi), && (\beta) \text{ and } (\gamma); \end{aligned}\right\} \qquad (64)$$

where

$$(k W_A^2 - W_1^2) \approx (k-1) W_1^2. \qquad (65)$$

Finally, a direct comparison between AND and OR systems yields

$$\log f_A = \log f_O + \log k - (k-1) \log (2\pi) + \tfrac{1}{2} D^2 U_1^2 (W_O^2 - W_A^2), \qquad (66)$$

where

$$k(W_O^2 - W_A^2) = 2kW_1 \sqrt{2/M_1} \left\{ G_1 (1 - 1/\sqrt{k}) + \epsilon + [G_1^2 (1 - 1/\sqrt{k})^2 - \epsilon^2] \Big/ W_1 \sqrt{2M_1} \right\}. \qquad (67)$$

These formulas reveal that:

1. The OR connection is preferable in conjunction with time-constant set α. This provides the smallest announcement delay and best reliability of the statistical theory on which all these equations are based.

2. The OR connection with sets β or γ yields no improvement over a single channel and is therefore ill-advised.

3. For sets β and γ, the AND connection results, in contrast, in a steady improvement with addition of more channels, which is about the same for set β or γ in comparison to a single channel. The overall false-alarm frequency is, however, very much smaller for set β.

These conclusions further suggest that a composite logic system, consisting of two stages of AND logic, can reduce the false-alarm frequency in connection with time-constant sets β and γ, by overcoming the time uncertainty inherent in the choice β such that the false-alarm frequency becomes that pertaining to this set, but with a gate $T_G = T$:

$$\log f''_A = B + k \log y. \tag{63e}$$

This implies the following additional conclusion:

4. For any transient-detection problem that can tolerate a precise announcement delay, a two-stage AND system can effect a very low false-alarm frequency, at a lower equipment cost than a multiple one-stage AND system. Detection problems in this category include scanning systems and fuel-failure monitoring systems complemented by direct-reading instrumentation. In the latter case, the transient detector provides confirmation, at enhanced sensitivity and reliability, of the verdict rendered by the other instrument.

APPENDIX A

An Alternative Derivation of the False-alarm Frequency

The false-alarm frequency, or crossing frequency of a given level in the upward direction was derived in Section IV on the basis of a heuristic argument, following Campbell and Francis.[19] To show that the same result is obtained from a mathematically more rigorous argument, we shall consider here the zero-crossing rate, a quantity related to the false-alarm frequency, by applying a number of equations derived in several of the standard texts cited in the list of references.[2-5,21]

The zero-crossing rate at zero level can be related in a straightforward way to the probability $P(xy < 0)$ that a trace sample, $x(t)$, taken at time t and another sample, $y(t+\tau)$, taken a small time τ later, have opposite signs, such that the trace must have crossed the zero (with respect to which sample height x is defined) an odd number of times.

This probability is formally derived from the correlation coefficient r for the joint distribution of two random processes. We suppose here that these processes are each described by Gaussian distributions, such that their joint distribution is given by

$$f(x, y; \sigma_x, \sigma_y, r) = \left(2\pi\sigma_x\sigma_y\sqrt{1 - r^2}\right)^{-1} \exp\left\{\left[\left(\frac{x}{\sigma_x}\right)^2 + \left(\frac{y}{\sigma_y}\right)^2 - 2\left(\frac{xy}{\sigma_x\sigma_y}\right) r\right] \Big/ 2(1 - r^2)\right\}. \quad (A.1)$$

It may now be shown that

$$P(xy < 0) = (\cos^{-1} r)/\pi. \quad (A.2)$$

For the foregoing situation, we further can write the correlation coefficient in terms of the autocovariance

$$R(\tau) = \int_0^\infty F(t)F(t - \tau)\, dt, \quad (A.3)$$

as

$$r = R(\tau)/R(0). \quad (A.4)$$

If we now consider a very small delay, τ, the probability of an odd number of crossings amounts to the single crossing probability, and even that is small enough to allow expansion of the cosine with retention of only the first two terms. Thus, we obtain the single-crossing probability

$$p_1 = (\sqrt{2}/\pi)[R(0) - R(\tau)]^{1/2}/\sqrt{R(0)}. \tag{A.5}$$

Now the autocovariance, $R(\tau)$, is an even function; hence, its expansion with respect to τ about the origin can be written

$$R(\tau) = R(0) + \tfrac{1}{2}R''(0)\tau^2 + \ldots . \tag{A.6}$$

Hence, for small τ, we obtain directly the zero-crossing frequency by inserting Eq. A.6 into Eq. A.5 and dividing by τ:

$$f_0 = (1/\pi)[-R''(0)/R(0)]^{1/2}. \tag{A.7}$$

The autocovariance, $R(0)$, given by Eq. A.3 with $\tau = 0$, is identical with the second moment of the level distribution calculated by Campbell's theorem. From this, we find

$$f_0 = (1/\pi)\left\{\int [F'(t)]^2\, dt \Big/ \int [F(t)]^2\, dt\right\}^{1/2}. \tag{A.7'}$$

Hence, the zero-crossing frequency is established as proportional to the ratio of the second moments of the level and slope distributions, respectively, which result was obtained earlier by a more direct argument.

The zero-crossing frequency can readily be shown to be the limiting case of the level-crossing frequency for any level above zero. Thus far, the direction of crossing has not been defined; the false-alarm frequency is evidently identical with the level-crossing frequency in the positive direction and hence comes to one-half the latter.

APPENDIX B

Effect of Linear Filtering on CRM Statistics

For ordinary purposes, the principal information delivered by a count-rate meter is a voltage or current proportional to the input rate, i.e., the first moment of the CRM trace distribution. In connection with transient monitoring, however, this information is of little interest, but the higher moments of the trace distribution, which determine the false-alarm frequency, are significant. It is thus generally advantageous to discard the first moment through a dc blocking capacitor. Together with the input resistance of the following circuit (the CR discriminator), this capacitor defines a third time constant for the system. In the body of this report, this time constant was assumed to be large in comparison with the other time constants and could thus be ignored. This assumption is now dropped in order to consider the effect of a linear filter featuring three arbitrary time constants, of which two are high-pass filters and one is a low-pass filter.

The response of such a network to a single input pulse is readily derived, and may be written most conveniently in terms of the half-power frequencies:

$a = 1/T_a$, where T_a = dc blocking final clipping time constant;

$b = 1/T_b$, where T_b = CRM time constant;

and

$c = 1/T_c$, where T_c = integrating (low-pass) filter time constant.*

The circuitry is assumed to isolate each of these filter stages from the others and thus forestall loading effects.

We first require the response H(t) of such a filter to a transient, of duration T and mean number of pulses M, valid for any time $t \geq T$, and further the response F(t) of the circuit to a single voltage pulse ΔV impressed on the CRM bucket condenser. For present purposes, we are moreover interested chiefly in a filter of equal frequency response $b = c$, where the final clipping lower cutoff frequency is at first arbitrary, and then will be specified to be zero (corresponding to the treatment in the text) or to be equal to the other frequencies, $a = b = c$.

*The designations a, b, and c, in preference to the usual symbols ω_a, ω_b, and ω_c, are used to avoid difficult-to-read and readily misprinted subscripts.

For the general case,

$$H(t) = -\frac{cM\Delta V}{T} \sum_{a,b,c} \frac{(e^{aT} - 1)\, e^{-at}}{(b-a)(c-a)}, \tag{B.1H}$$

and

$$F(t) = c\Delta V \sum_{a,b,c} \frac{ae^{-at}}{(b-a)(c-a)}, \tag{B.1F}$$

where the summation sign indicates all cyclic permutations.

Now, specifying $b = c$, one finds

$$H'(t) = M\Delta V\left(\frac{b}{b-a}\right)^2 \left\{e^{-bt}\left[\frac{e^{bT} - 1}{bT}[1 + (b-a)t] - \frac{b-a}{b}\, e^{bT}\right] - \frac{e^{aT} - 1}{bT}\, e^{-at}\right\}, \tag{B.2H}$$

and

$$F(t) = \Delta V \frac{b}{b-a}\left[bte^{-bt} - \frac{a}{b-a}(e^{-at} - e^{-bt})\right]. \tag{B.2F}$$

Let $a = 0$; then Eqs. B.2H and B.2F become

$$H_0'(t) = M\Delta V e^{-bt}\left[\frac{e^{bT} - 1}{bT}(1 + bt) - e^{bT}\right], \tag{B.3H}$$

and

$$F_0'(t) = \Delta V e^{-bt} bt. \tag{B.3F}$$

Similarly, for the special choice $a = b = c$,

$$H_b'(t) = (M\Delta V/bT)[e^{-bt}(bt)^2/2 - e^{-b(t-T)}\, b^2(t-T)^2/2], \tag{B.4H}$$

and

$$F_b'(t) = \Delta V e^{-bt} bt(1 - bt/2). \tag{B.4F}$$

These equations are readily derived from circuit theory and need no further comment. We shall use them here to calculate the peak response to the transient, H_{max}, as well as the second moment of the trace distribution and the trace slope distribution, from which we may infer the zero-crossing rate, according to Appendix A. The first correction term of the Edgeworth series (discussed in Section V), another parameter influencing the statistical quality of transient detection, will then be calculated. The full effect of the filter can then be estimated.

The peak response for $b = c$, $a = 0$ had already been calculated; it comes to

$$(H_0')_{max} = M\Delta V(hy)^{-1} \exp[-(h-1)\,y], \tag{B.5}$$

where we introduce the notation

$$h = (1 - e^{-y})^{-1} \tag{B.6}$$

and

$$y = bT. \tag{B.7}$$

For $a = b = c$, we differentiate Eq. B.4H and equate the result to zero, thus obtaining a quadratic equation for the peak time. (Equation B.4H describes a trace response with a maximum and a minimum, similar to a doubly differentiated (DD-2) pulse-amplifier signal.) The maximum occurs at the time

$$t_m = b^{-1}(1 + yh - Z), \tag{B.8}$$

where

$$Z = [1 + h(h-1)\,y^2]^{1/2}. \tag{B.9}$$

Inserting Eq. B.8 into Eq. B.4H and rearranging, we find that

$$(H_b')_{max} = (H_0')_{max}(Z-1)\exp(Z-1) \tag{B.10}$$

is the peak response for this special case. The factor $(Z-1)\exp(Z-1)$ can be readily estimated. An expansion of the root in Eq. B.9 yields

$$Z = \sqrt{2}(1 - y^2/24 + y^4/480 - \ldots)^{1/2}, \tag{B.11}$$

which clearly varies only slightly for values of the parameter y within the range of practical interest, $0 \le y \le 1$. Thus, to a reasonable approximation, we may put

$$(H_b')_{max} \approx 0.626(1 - 0.101y^2)(H_0')_{max}. \tag{B.10'}$$

The second moment of the distribution generated by random input at rate n in a filter b = c, a = 0, has also been calculated before; it comes to

$$\lambda_2' = (\sigma_0')^2 = n\Delta V^2/4b. \tag{B.12}$$

A similar calculation, applying Campbell's law to Eq. B.1F, yields the second moment for three generally different filter frequencies a, b, and c:

$$\lambda_2 = \sigma^2 = n\Delta V^2 c^2/2(a+b)(a+c)(b+c). \tag{B.13}$$

The special case, a = 0, given by Eq. B.12, follows directly from Eq. B.10; so also does the case a = b = c, for which

$$(\sigma_b')^2 = n\Delta V^2/16b. \tag{B.14}$$

The S^2/N ratio can thus be found for the latter case, using Eqs. B.10' and B.14. It amounts to

$$R_b' = 1.252R_0'(1 - 0.1y^2), \tag{B.15}$$

in terms of the a = 0(S/N) ratio R_0'. Thus far, the effect of a short final clipper in comparison to a very low final frequency cutoff ($a \approx 0$) is a certain loss of signal, i.e., transient response, accompanied by a somewhat more severe reduction in the rms fluctuations, which in fact are halved. A plot of the exact response function, shown in Fig. B.1, reveals that for $y > 1$, R_b' rather quickly drops below R_0'. This does not alter the fact that $R_b' > R_0'$ within the range of practical interest.

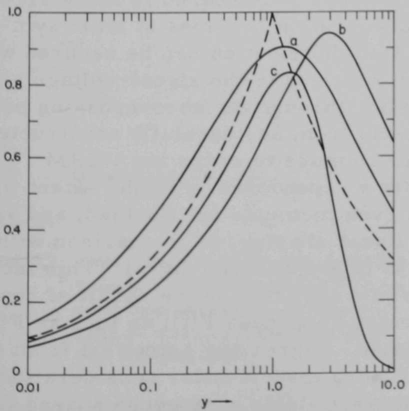


Fig. B.1

Filter Response to a Transient of Duration T. Curve a: Single high-pass RC element (Eq. 12). Curve b: One low-pass, one high-pass, filter (Eq. 20). Curve c: Two high-pass, one low-pass, elements (Eq. 23). Cutoff frequency of all elements is $b = Tb^{-1}$. Broken line shows response of digital CRM with "memory" T_b (Eq. 24). The normalized response (unit signal and background) is plotted against the parameter y = bT; vertical scale is linear to emphasize peak region detail. (A similar plot with logarithmic scale is provided in Fig. 4.)

113-953

The zero-crossing frequency is given by Eq. A.7. Differentiating Eq. B.2F and solving the integral giving the mean square of the slope of the pulse response, or slope autocorrelation at zero delay, one finds, after some rearrangement,

$$\int (dF/dt)^2 \, dt = \Delta V^2 b^2 (b + 2a)/4(a + b)^2 \qquad (B.16)$$

for $b = c$, with arbitrary final clipping frequency a. From this, the zero-crossing frequency comes to

$$f_0 = (b/2\pi)(1 + 2a/b)^{1/2}. \qquad (B.17)$$

Comparing now $a = b$, and $a = 0$, the zero-crossing frequency evidently increases by a factor of $\sqrt{3}$ for sharp clipping. This effect, which makes itself felt as increased input noise in pulse amplifiers that feature DD-2 filtering, is hardly surprising. From the point of view of transient detection, this effect is bad news, but may be more than compensated for by the increased exponential factor (for $y \approx 1$).

We shall now consider, finally, the effect of clipping on the distribution and, more particularly, consider how filtering can make this distribution more symmetric with respect to the mean trace level. The deviation of an actual distribution from symmetric shape finds its expression in those terms of the Edgeworth series that are proportional to odd powers of the difference between output signal and mean output signal. These terms also are multiplied by a "skewness" parameter, which is proportional to an odd moment of the distribution.

In principle, the response F(t) can always be shaped to make all odd moments vanish, whereupon the trace distribution becomes at least symmetric, if still not entirely Gaussian. This distribution can be secured with any F(t) that exactly repeats itself, with reversal of the signal-voltage sign, such that F(t) folds into itself upon rotation through the zero-crossing point by π. (Such response shapes are available from any carefully constructed DD-2 pulse amplifier, and can therefore be made to order for a CRM channel, with a suitable scaling of the time dependence.) To the extent to which Campbell's theorem is valid, all even moments are doubled, and all odd moments canceled, by this type of linear shaping, in comparison with the distribution that would result from only one-half the signal. This fact is intuitively plausible if one considers that the trace is the result of randomly superposing a large number of pulse responses F(t); as long as F(t) is symmetric, the trace is also symmetric. Digressing somewhat from the task of investigating the simple final clipping circuit under consideration, we may suppose that a filter is available that yields a response shaped like a single period of a sine wave, $F(t) = \Delta V \sin(\omega t)$. The second moment of the trace made by a random input with such a filter is then

$$\lambda_2 = (n\pi/\omega)\ \Delta V^2,$$

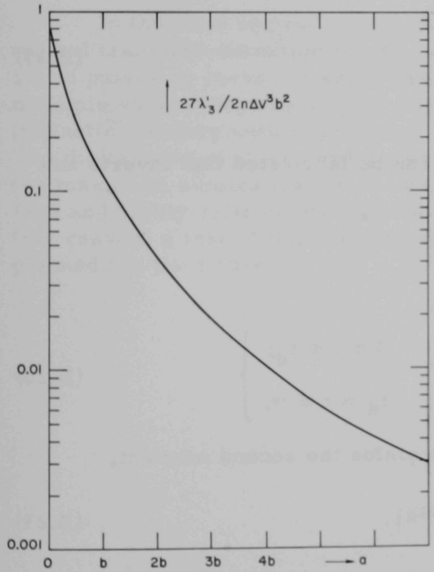
the zero-crossing frequency can readily be found to be

$$f_0 = \omega/2\pi,$$

and the third moment vanishes, as is evident from inserting this pulse shape in Campbell's equation and integrating.

Proceeding now to evaluate the third moment for the elementary circuit described above, we take the response given by Eq. B.2F with $b = c$ and a arbitrary. Integration and considerable rearrangement yields

$$\lambda_3 = \frac{2n\,\Delta V^3 b^2}{27}\,\frac{8(a+b)^2 - 5ab}{(a+2b)^3(2a+b)^2}. \tag{B.16}$$



113-654

Fig. B.2. Third Moment of Distribution, According to Eq. B. 16. The ordinate is proportional to the third moment; the abscissa indicates the lower cutoff frequency of the final high-pass filter.

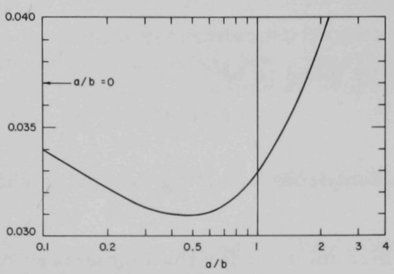
Equation B.16, plotted in Fig. B.2, reveals an impressive decrease of the third moment with increasing lower cutoff frequency a. However, the skewness parameter,

$$S = \lambda_3/(3!)\lambda_2^{3/2},$$

is actually not strongly affected by final clipping. From Eq. B.13, we find the second moment for $b = c$. Hence,

$$S = \frac{8}{81}\left(\frac{b}{n}\right)^{1/2}\frac{8(a+b)^2 - 5ab}{(2a+b)^2}\left(\frac{a+b}{a+2b}\right)^3. \tag{B.17}$$

Equation B.17 is plotted in Fig. B.3. The skewness of the distribution can apparently be worsened by very strong final clipping. In the light of the above remarks, the response shape exhibits relatively the best symmetry for $a = 0.5b$, but even at that point is not sufficiently symmetric to affect distribution skewness significantly. This circumstance suggests that a trace symmetrization filter will have to be designed carefully, to match positive and negative excursions precisely.



113-659

Fig. B.3

Skewness Parameter S versus Ratio of Time Constants a/b, in Arbitrary Units. The skewness does not increase without limit as a/b increases, but eventually approaches a value just twice the lower (a/b = 0) limit indicated on the left margin.

For the special cases a = 0 and a = b = c, we find the skewness parameters

$$S_0' = 0.0987\sqrt{b/n}, \tag{B.18}$$

and

$$S_b' = 0.889S_0' = 0.0879\sqrt{b/n}, \tag{B.19}$$

respectively.

Suppose, however, that a delay can be fabricated that inverts and repeats the pulse,

$$F(t) = (\Delta V/2)\, bte^{-bt},$$

after some time t_d. We have, then,

$$\left.\begin{aligned} F(t) &= (\Delta V/2)\, bte^{-bt}; & 0 \le t \le t_d; \\ &= (\Delta V/2)\, b(t - t_d)\, e^{-b(t-t_d)}; & t_d \le t \le \infty. \end{aligned}\right\} \tag{B.20}$$

The usual treatment by Campbell's law yields the second moment,

$$\lambda_2 = (n\Delta V^2/8b)[1 + 2(1 - bt_d)\, e^{-bt_d}], \tag{B.21}$$

as well as the third moment,

$$\lambda_3 = (n\Delta V^3/72b)\{2(bt_d - 1)\, e^{-bt_d} + [3bt_d(bt_d - 1) - (bt_d - 1) + 1]\, e^{-2bt_d}\}. \tag{B.22}$$

In the limit of large delays, the second moment is twice its value for an unrepeated pulse,* and the third moment vanishes. On the other

*Of height $\Delta V/2$, for comparison.

hand, the choice $bt_d = 1$ also doubles the second moment, but leaves a small residual third moment, $\lambda_3 = n\Delta V^3/72e^2b$. The skewness parameter for this case comes to $0.0071(b/n)^{1/2}$, which is more than an order of magnitude smaller than the skewness with optimized double differentiation, although the actual pulse shape is not markedly different for these two filters. This illustrates the sensitivity of the odd moments (hence, skewness) to pulse-shape symmetry.

The third term in Edgeworth's expansion is again entirely composed of odd moments. A pulse shape that makes these moments either very small or zero thus allows in principle a somewhat greater reliance on the correction, which is then largely due to the fourth moment. This stratagem therefore provides a more reliable estimate of the false-alarm frequency (as well as a reduction by a worthwhile amount in the actual false-alarm frequency).

In the time regime defined by the expected transient duration for a typical transient-detection system, e.g., the fuel-failure monitor, the shaping of pulses by means of delay lines is difficult and expensive, requiring multiple sonic delays or possibly readin and readout on a disc or tape magnetic memory with appropriate spacing between write and read heads. The digital system described earlier, although less flexible with regard to the maximum admissible count rate, appears as a relatively less expensive and highly reliable alternative means of precise pulse shaping. For this reason, a test of this possibility through a computer simulation is planned for the future.

APPENDIX C

A Note on an Expansion Used in This Report

The response of a filter with two equal time constants T_b to a transient of strength M and duration T is given by

$$R = \frac{2M}{\sqrt{N}} \frac{1 - e^{-y}}{\sqrt{y}} \exp \frac{-y}{e^y - 1} = \frac{M}{\sqrt{N}} U_1(y), \tag{C.1}$$

where

$$y = \frac{T}{T_b}$$

and

$$N = nT = \text{background (mean) during transient.}$$

The exponent $-y/(e^y - 1)$ in the above equation can be expressed in a series of Bernoulli as follows:

$$\frac{y}{e^y - 1} = 1 - \frac{y}{2} + \frac{B_1 y^2}{2!} - \frac{B_2 y^4}{4!} + \frac{B_3 y^6}{6!} - \dots, \tag{C.2}$$

with

$$B_1 = 1/6,$$

$$B_2 = 1/30,$$

$$B_3 = 1/42,$$

$$\vdots$$

Generally,

$$B_k = 2(2k)! \sum_{j=0}^{\infty} (2\pi j)^{-2k}.$$

Inserting Eq. C.2 into Eq. C.1 and rearranging yields

$$U_1(y) = (4/e\sqrt{y}) \sinh (y/2) \exp\left[\sum_{k=1}^{\infty} B_k y^{2k} (-)^k/(2k)!\right]. \tag{C.3}$$

For small values of y, the Bernoulli series is also small in value; hence, the exponential term may be dropped. For values of y near unity, one may expand the hyperbolic sine and obtain

$$U_1(y) = (2\sqrt{y}/e)[1 - y^2/24 + 1.1(y^2/24)^2 - 1.15(y^2/24)^3 + \ldots]. \qquad \text{(C.4)}$$

The second and higher terms are quite small, even for y values somewhat larger than unity. The series can still be used without significant loss of accuracy for still larger values, provided more terms are included. This makes Eq. C.4 useful in conjunction with digital computation of a number of problems in which equations similar to Eq. C.1 occur, in view of the wide use of equal time-constant pulse processing.

APPENDIX D

Statistical Performance of Digital Count-rate Meters

The false-alarm frequency for transient detection is treated in the body of this report on the basis of the continuous level and slope distributions obtained with analog equipment. Digital count-rate meters, in contrast, deliver an output consisting of small but finite steps, and thus required discrete distributions for the development of false-alarm-frequency formulas.

The fact that digital circuits, based on certain logical connections of elementary "go/no-go" units, offer almost absolute long-term stability has been recognized for some time. Only recently, however, with the advent of integrated (single-chip) digital units, has the cost of constructing an extensive logical network (required to provide the memory feature characteristic of count-rate meters) become low enough to make digital count-rate meters a practical possibility. These developments have similarly influenced the cost and availability of computers. This discussion of the statistics of digital count-rate metering may thus apply equally to a fully integrated instrument, or to a computer program through which similar basic circuit elements, incorporated in a computer, perform exactly the same function. As mentioned in the body of this report, the choice between on-line computation and a separate instrument for a given transient monitor depends on a number of cost factors which are expected to vary greatly in the near future. A realistic appraisal must balance the relatively high cost of software and possible interfacing against the flexibility, availability for other tasks, and large memory of the computer, and consider, on the other hand, the advantage of compactness and direct accessibility for maintenance and testing of the developed instrument. The choice becomes even more difficult when one considers the possibilities of logical and/or multiplexing of several independent detector channels, as outlined in Chapter VII. Thus, a number of relatively simple digital circuits, employed in such a logic network, may well outperform a more sophisticated single digital count-rate meter in some particular transient-detection setup.

To provide at least the framework for the task of optimizing digital processing equipment for a certain transient monitor, we shall derive false-alarm formulas for an elementary count-rate measurement device (i.e., a cyclic count-and-dump scaler) as well as for an instrument that can be logically developed from a number of such channels and includes a memory feature.[16] Other more complex digital count-rate meters have been described in the literature.[16] These instruments aim at the simulation of the performance of analog circuitry by providing the digital equivalent of a condenser discharge; their false-alarm frequency should thus be about the same as pertains to analog instruments (to the extent that the simulation is realistic).

A cyclic count-and-dump scaler, sometimes referred to as a "digital count-rate meter," is, strictly speaking, an instrument of a somewhat different kind: information is delivered only at intervals and necessarily concerns a time average over the immediate past count interval, T_c, beyond which there is no memory. In a transient monitor with a quasi-constant background input rate n and an expectation of M signal pulses within a short time span T, it is readily apparent that a very long counting interval will tend to bury the signal in a large background component $nT_c = N$, much as is the case for analog circuitry treated in Chapter III. A counting interval comparable to T, on the other hand, involves the risk of splitting the signal between two intervals, such that neither count is sufficient to trip the alarm level (a preset number of counts A, where $N < A < M$). This risk is lessened by feeding the input into two parallel channels, cycled out of phase, one of which is then more likely to contain the full number M within a single interval T_c. At the same time, the false-alarm frequency is necessarily doubled for this arrangement, while the mean announcement delay is shortened. Further development of this stratagem leads to a system consisting of M channels, for which the risk of missing the transient becomes very small as M is made large, while the false-alarm frequency is commensurately increased. This kind of system is obviously unwieldy and may be replaced with a unit consisting of a shift register containing M bits and an add-subtract scaler, whose performance is described in Chapter III (or, in more detail, in Ref. 16). Each pulse is added to the scaler and also entered in the shift register. At the end of an interval $T_b = M/r$, where r equals the shifting rate, the pulse emerges from the other end of the shift register and is subtracted. In comparison to an elementary analog count-rate meter, this system produces a step increase of the output voltage which, instead of decaying exponentially, lasts at full height for a time T_b, and then ceases abruptly. The second moment of the output trace can thus be computed from Campbell's theorem with a "square-pulse" single-input response,

$$
\begin{aligned}
F(t) &= 1, \; 0 \le t \le T_b; \\
&= 0, \; t < 0 \text{ and } t < T_b. \qquad \text{(D.1)}
\end{aligned}
$$

The slope distribution, on the other hand, is evidently not derivable from the time derivative of Eq. D.1, as will be considered further on.

We shall now find the false-alarm frequency for a count-and-dump channel, on the basis of a simple probabilistic model which assumes only that the input rate n is Poisson-distributed. This assumption implies a mean scaler content $N = nT_c$ and a variance also equal to N. The probability of finding K counts, W_K, is given by

$$
W_K = (N^K/K!)e^{-N}. \qquad \text{(D.2)}
$$

The probability of occurrence of a false alarm thus comes to the sum

$$P_f(A) = \sum_{K=A}^{\infty} W_K. \tag{D.3}$$

To find a false alarm within some selected time interval T_c, it is necessary that the count just preceding this interval does not trip the alarm, while the next count does. The false-alarm frequency, f, is thus the product of the adjoint probabilities for these conditions, divided by the inspection interval T_c; that is,

$$f = \sum_{K=0}^{A-1} W_K \sum_{L=A}^{\infty} W_L / T_c. \tag{D.4}$$

It is expedient to express Eq. D.4 in a more convenient form. On the assumption that K is large, we make use of Stirling's theorem, through which the probability W_K becomes

$$W_K = \left(e^{K-N}/\sqrt{2\pi K}\right)(N/K)^K. \tag{D.5}$$

Introducing the parameter

$$\beta = (K - N)/\sqrt{N} \tag{D.6}$$

and taking logarithms, one finds that

$$\log W(\beta) = \beta\sqrt{N} - \tfrac{1}{2} \log 2\pi N - (N + \beta\sqrt{N} + \tfrac{1}{2}) \log (1 + \beta/\sqrt{N}). \tag{D.7}$$

The last term of Eq. D.7 is expanded and small quantities are dropped, whereupon one obtains the Gram-Charlier or Edgeworth representation of the Poisson distribution, in terms of a Gaussian and a correction series already discussed in detail in Chapter VI:

$$W(\beta) = (2\pi N)^{-\frac{1}{2}} e^{-\beta^2/2}[1 + (\beta^3 - 3\beta)/6\sqrt{N} + \dots]. \tag{D.8}$$

Proceeding now to find an expression for the false-alarm frequency, we replace the sums in Eq. D.4 with integrals, approximate large limits by ∞, and neglect 1 in comparison with A. Moreover, we may drop the Edgeworth series for purposes of obtaining merely a reasonable approximation, and thus obtain the simple expression

$$f = (\tfrac{1}{4}T_c)[1 - \Phi^2(U)], \tag{D.9}$$

where

$$U = (A - N)/\sqrt{2N} \tag{D.10}$$

and Φ is the error function, introduced in Chapter IV. We note that Eq. D.9 predicts a "zero-crossing" frequency (U = 0), which is directly evident if one considers that there is an even chance for crossing, or not crossing, the mean N between any two randomly chosen intervals; another factor of 2 is obtained when crossing is counted in only one direction.

For large values of the argument U, on the other hand, Eq. D.9 may be expressed in a suitable expansion which converges fairly rapidly,

$$f = [n/2\pi T_c(A - N)^2]^{\frac{1}{2}}[1 - N/(A - N)^2 + \ldots]e^{-(A-N)^2/2N}. \tag{D.9'}$$

Equation D.9' shows a particularly steep decline of the false-alarm frequency with level above mean, A - N, and with the count interval T_c. This performance is so good that it suggests the practical possibility of two out-of-phase channels, as considered above.

We turn now to the false-alarm frequency of an assembly of M such count-and-dump channels, arranged to cycle at evenly spaced intervals, which also applies to the digital count-rate meter described in Ref. 16, or to an analog count-rate meter that develops square pulses. This false-alarm frequency is evidently smaller than M times the rate obtained from Eq. D.9', since a false alarm, when it occurs, will probably appear in several of the M channels, yet such an alarm should be counted only once. We shall use the digital count-rate meter as a basic model for developing a false-alarm-frequency formula, and note that the number of pulses stored in the shift register at any time must always be exactly equal to the number appearing in the add-subtract scaler. The situation we are considering has a strong resemblance to certain aspects of Queuing Theory, a subject covered by "an incredibly voluminous literature," to quote a remark by Feller.[4] A search of this literature for the solution of the exact problem posed here turned out, nevertheless, to be unavailing. The development given below is therefore not based on previously published work--admitting the possibility that a more exhaustive search may discover a treatment of a closely similar problem in traffic control, design of a servicing facility, or telephone communications.

We shall assume that the number of available shift register bits M is considerably larger than the mean number N of occupied bits. The system thus has a negligible digital dropout; that is, the chance that two inputs arrive within the shifting period $1/r$ is very small. (For a practical unit, such a second pulse can be temporarily stored; hence the digital dropout is a manageable problem even at high input rates, as discussed in Ref. 16.) For every shift, the system plays a coin-tossing game, with an a priori probability n/r of "success" and adjoint probability $1 - n/r$ of "failure." Hence, the probability of finding exactly K pulses stored in the shift register and therefore also in the scaler) is given by a binomial rather than a Poisson distribution law,

$$P_M(K) = \{M!/[(M-K)!\,K!]\}(N/M)^K[1-(N/M)]^{M-K}. \tag{D.11}$$

(Note that $n/r = N/M$.)

A false alarm occurs whenever the store, upon the completion of a game, happens to contain exactly A - 1 events and the next game results in adding another event. This further implies that the game played T_d earlier had a negative result, such that the shift register does not lose one event at its delivery end while it acquires a new one at its receiving end. Finally, the inspection interval for this situation has been shortened to T_d/M, whence the false-alarm frequency comes to

$$f = (M/T_d)[1-(N/M)](N/M)P_{M-1}(A-1)$$

$$= (A/T_d)[1-(N/M)]P_M(A). \tag{D.12}$$

As before, this can be readily processed by Stirling's formula and the Taylor-McLaurin expansion. We further introduce the alarm-level parameter

$$A = N + \beta\sigma, \tag{D.13}$$

where

$$\sigma = \sqrt{[1-(N/M)]N} \tag{D.14}$$

is the variance of the binomial distribution given by Eq. D.11. After some manipulation, we obtain the Edgeworth representation,

$$f = [1-(N/M)](N+\beta\sigma)\left\{e^{-\beta^2/2}\Big/T_d(2\pi\sigma)^{\frac{1}{2}}\right\}$$
$$\times\left\{1+(\beta^3-3\beta)[1-(2N/M)]/6\sigma^2+\dots\right\}. \tag{D.15}$$

Dropping small quantities, we obtain

$$f \approx \sqrt{n/2\pi T_d}\; e^{-\beta^2/2};$$

$$\beta \approx (A-N)/\sqrt{N}. \tag{D.16}$$

As already suggested, this false-alarm frequency is considerably higher than that for a single count-and-dump channel, given by Eq. D-9. On the other hand, the detection probability for the M-channel instrument is, significantly improved.

Equation D.16 can be derived also on the basis of the square-pulse count-rate meter model, for which we may take the argument of Campbell and Francis, as expressed in Eq. 28:

$$f = P(a) \int_0^\infty sP(s)\,ds, \qquad \text{(D.17)}$$

where

$$P(a) = e^{-\beta^2/2} \Big/ \sqrt{2\pi\sigma^2} \qquad \text{(D.18)}$$

is the level distribution probability. The slope distribution, $P(s)$, is given by a sum of delta functions. Realistically supposing that the square pulses actually have trapezoidal shape with constant rise and fall slope s_0, we can put

$$P(s) = b[\delta(s - s_0) + \delta(s + s_0)] + (1 - 2b)\,\delta(s). \qquad \text{(D.19)}$$

The weighting constant b is the probability that a random sampling of the trace will catch the trace while rising or falling,

$$b = n/s_0. \qquad \text{(D.20)}$$

The second moment of the level distribution may be calculated with response $F(t)$ given by Eq. D.1, to a sufficient approximation; this yields simply

$$\sigma^2 = nT_d. \qquad \text{(D.21)}$$

After inserting Eqs. D.18-D.21 into Eq. D.17 and integrating over all positive slopes, one finds a false-alarm frequency given exactly by Eq. D.16.

To sum up, false-alarm formulas can be readily derived for digital count-rate meters and/or simple count-dump cyclic scalers. The latter formulas may then be modified to find the false-alarm frequency for several out-of-phase cyclic scalers, and either type of count-rate processor can further be deployed in an AND/OR logic system deriving input from several statistically independent channels. The entire system can be constructed as a separate instrument or as a computer interface, or produced entirely through programming. These possibilities offer a wide range of choices; indeed, the optimization of such a system itself appears to require extensive programming of a computer. As in other applications of systems analysis, this process may be expedited by programming a simulation of the problem instead of developing solutions to equations that (as frequently stressed in this report) tend to break down when certain limits are reached by the input parameters. Such a simulation program offers the further advantage that it can readily be adapted to on-line processing of a surveillance or scanning instrument. The cost of this investigation must be weighed against the potential cost of failure of a surveillance system. In that connection, it may be emphasized once more that it is not enough to maintain all instrument components of such a system in excellent operating condition, but that time constants and alarm levels (i.e., the detection strategy) must be optimized as well in order to provide the best margin of safety.

APPENDIX E

Information Processing and Display System Design

In connection with the general description of transient detection systems of various types presented in Section II, some of the means of displaying fast transients are described in somewhat more detail here, to illustrate the strong influence of input parameters on equipment design.

For most of the vehicular area-survey systems, the duration of the transient may amount to at least several seconds. Hence a conventional count-rate meter, filter circuit, and pen recorder are entirely adequate and probably preferable to more elaborate processors. Several channels can be readily combined, possibly along lines suggested in Section VI, or several channels featuring different time constants employed in parallel. The stability of the equipment can be checked against a standard source at suitable intervals, or a feedback stabilizing system devised.

Survey systems operating under conditions where the time consumed by a survey must be minimized (while no limitations exist on the sweep speed) may yield transients that are substantially shorter than the response of pen recorders. Such equipment is nowadays frequently designed for computer control. Hence, processing can be most efficiently carried out in the digital regime, through suitable programming. The program must provide a digital count-rate meter routine for each input channel, with certain time constants, warning levels, and other parameters. Display, either continuous or on demand, can be provided by a cathode-ray tube. Signal enhancement can be effected where this is useful; for other purposes, as, for example, certain track-scanning tasks, it may be sufficient to write a signal-recognition program such that the computer calculates and/or plots the number of tracks per unit area without on-line display.

The processing of transients from a fuel-failure monitor appears to be intermediate between the two above-mentioned applications so far as transient duration is concerned. Chart recorders of the potentiometric feedback type, with a typical response time of 1 sec, are clearly incapable of displaying fractional-second transients, while the great speed of computer processing is wasted on channels whose normal count rate is only 10-100/sec. At the same time, continuous duty and immediate display of any transient are required, while input information which is clearly established as background only can be discarded after some storage period. A permanent record is wanted only when a transient occurs.

All these features can be provided by a computer, with a digital processing program and oscilloscope display. Permanent display is available by means of tape or digital printout, chart or graph-writing equipment or photograph of the oscilloscope: the latter can be taken on command of an alarm discriminator, while the other, slower outputs can be obtained at any later time, assuming that erasure of the transient, stored in the memory in digital form, is also inhibited by the alarm. The cost of such a system is, however, rather high. Where a large computer already exists for purposes of processing other instrument channels, the additional cost of memory capacity, interfacing and software for the fuel failure channels may be modest in comparison with the cost of the whole system. When such computer capacity is not provided, (as was the case for EBR-II), a specially designed information processor may be a reasonable solution. Such a system, described in detail in Ref. 23, was originally designed for the FERD loop.

In that processor, incoming counts are directly written on magnetic tape and also fed to a count-rate meter with attached alarm discriminator. After a certain delay, the tape passes over a readout head and thence into a magazine with about 5 min maximum storage capacity. Upon emerging from the magazine, the record is erased just ahead of the write head. If an alarm is given, the paper feed and light intensity of a fast light-writing chart recorder are turned on, such that the written record starts 1-2 sec ahead of the transient and then displays the whole transient. This readout can be repeated a number of times by interrupting both erasure and input and thus preserving the record. The whole system operates on two independent channels.

This relatively simple and inexpensive device allows recording at a paper feed rate which (if left on continuously) would consume several miles of paper per day, yet is required for a clearly recognizable record of fractional second effects. It also provides limited storage of transients in digital form. Its chief drawback appears to lie in the difficulty of securing continuous operation with minimum maintenance: presently available magnetic tape tends to wear badly in a continuous loop magazine and requires occasional replacement. Improved tape quality, which now appears to be a strong possibility, or mechanical design improvements which reduce wear should overcome this difficulty. On the other hand, a digital delay system, using shift registers which have recently come down considerably in cost, may well be preferred. This system would be employed in conjunction with a digital processor, of a total cost intermediate between that of a tape/analog unit and that of a computer.

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